

# Variable Stiffness Actuators: the user's point of view.

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**Abstract**—Since their introduction in the early years of this century, Variable Stiffness Actuators (VSA) witnessed a sustained growth of interest in the research community, as shown by the growing number of publications. While many consider VSA very interesting for applications, one of the factors hindering their further diffusion is the relatively new conceptual structure of this technology. In choosing a VSA for his/her application, the educated practitioner, used to choosing robot actuators based on standardized procedures and uniformly presented data, would be confronted with an inhomogeneous and rather disorganized mass of information coming mostly from scientific publications.

In this paper, the authors consider how the design procedures and data presentation of a generic VS actuator could be organized so as to minimize the engineer's effort in choosing the actuator type and size that would best fit the application needs. The reader is led through the list of the most important parameters that will determine the ultimate performance of his/her VSA robot, and influence both the mechanical design and the controller shape. This set of parameters extends the description of a traditional electric actuator with quantities describing the capability of the VSA to change its output stiffness.

As an instrument for the end-user, the VSA datasheet is intended to be a compact, self-contained description of an actuator that summarizes all the salient characteristics that the user must be aware of when choosing a device for his/her application.

At the end some example of compiled VSA datasheets are reported, as well as a few examples of actuator selection procedures.

**Index Terms**—Soft Robotics, Variable Stiffness Actuator, Variable Impedance Actuation, physical Human-Robot Interaction.

## I. INTRODUCTION

“True progress is that which places technology in everyone's hands.” Many expect that the first generation of robots fulfilling Henry Ford's vision will be one coexisting and physically cooperating with people, being capable of natural motions and much closer to human performance than today's robots. Future

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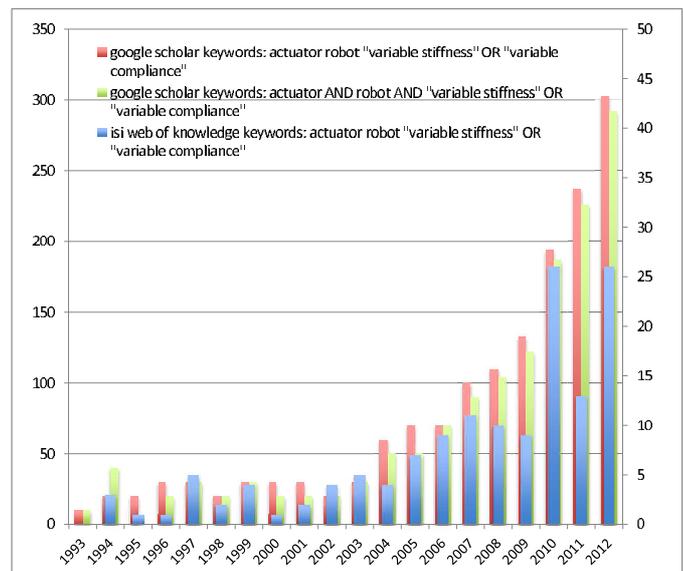


Fig. 1. Number of papers published on the topic of Variable Stiffness Actuation from 1990 to 2012, based on Internet databases. Sources and keywords are detailed in the caption.

robots are expected to be intrinsically safe, in the sense that interacting with them should not constitute a higher injury risk to humans than the interaction with another cautious human. In short, robots should move towards becoming a companion in everyday life. This requires that robots with similar size and mass as the humans also have comparable power, strength, velocity and interaction compliance. However this ambitious goal can hardly be achieved with the existing robot technology, in which the robots are designed primarily as rigid position or torque sources and most interaction skills are imposed by virtue of control software.

In recent times, Variable Impedance Actuation (VIA) has been proposed as a possible answer to fulfill some of these specifications. Among VIA, Variable Stiffness Actuators (VSAs) were the first solution to be investigated. A large number of VSA prototypes has been proposed by research centers all over the world over the past years, and novel solutions implementing the VSA concept are still being presented nowadays in every major conference. While a survey and discussion of the state of the art is out of the scope of this paper (the reader is referred to e.g. [Vanderborgh et al., 2013]), we limit ourselves here to few notes on the trend. While the idea of changing joint impedance through agonist-antagonist actuators

is deeply entrenched in robotics literature since very early years (see e.g. [Laurin-Kovitz et al., 1991]), the first VSA prototype for safe and fast motion control dates back to 2003 [Bicchi et al., 2003]. Only one paper on VSA was presented at ICRA 2004 [Hurst et al., 2004], but four full dedicated sessions on VSA were aired at ICRA 2011. The growth of VSA-related literature is shown in Fig. 1.

One of the earliest reasons behind the development of Variable Stiffness has been *safety*. The *safe brachistochrone* problem is the Optimal Control problem of minimizing the time needed to move a mechanical load from one position to another, while containing the danger level of a potential impact below a critical injury level. As shown in works such as [Bicchi and Tonietti, 2004], VSAs offer the possibility to move the load faster and more safely than other solutions based both on rigid or flexible joints.

Later studies considered the possibility of rapidly adjusting the transmission stiffness as a way to maximize some figure of merit of the task execution itself. An example, somehow dual to safety, is the maximization of impact energy (see [Haddadin et al., 2011] and [Garabini et al., 2011]). Simple tasks, such as kicking a ball or hammering a nail, require the link of a robot to build up kinetic energy and transfer such energy to a target by hitting it. Both robustness and performance considerations hint to the superiority of flexible joint robots to traditional rigid ones. Nevertheless, once again, the solution of the related Optimal Control problem, proposed in [Garabini et al., 2011], shows that VSAs can outperform fixed mechanical compliance systems such as Series Elastic Actuators by as much as 30%.

The aforementioned aspect of robustness, as well as the dependability of the system, are accurately discussed in works as [Haddadin et al., 2008], [Filippini et al., 2008], [Wolf et al., 2011]. On one side, the presence of compliant elements in a robot extends the life of the gearboxes by reducing the amplitude of stress peaks derived by impulsive dynamic phenomena (e.g. accidental contacts), while, on the other side, the intrinsic actuation redundancy present in some VSAs (those built with Antagonist elastic elements) increases their reliability in case of mechanical failures.

Finally, another interesting aspect of VSA systems is the intrinsic combination of energy efficiency and versatility. Studies such as [Visser et al., 2010], [Visser et al., 2011] and [Ozawa and Kobayashi, 2003], show how a VSA can be controlled to match their natural oscillations with a cyclic motion pattern in order to minimize the energy input in the system, *de facto* embodying a desired behavior in the mechanical properties of the system.

Despite these recent advancements introduced in the state of the art, the vast majority of the robotic community suffers from the lack of actuation units which can rival the functional performance of biological muscles. Fields as Service robotics, Walking and Humanoid robotics [Hurst et al., 2004] [Van Ham et al., 2007], Haptics and other Physical Human Robot Interactions as Orthotics [Blaya and Herr, 2004], Prosthetics and Rehabilitation robotics, moreover Bio-mimetic robotics and, last but not least, Versatile Manufacturing, Machining and Assembly [Kim et al., 2011] are ready, and even

now starting, to integrate the potential of VSA in their designs. This will really let robotics fill one of the gaps that still keeps it from the development of machines which can match the strength, deftness, velocity, efficiency and versatility shown by natural biological systems and, in particular, by humans.

This work intends to organize the vastness of the VSA state of the art by establishing a common language for VSA designers to confront one another and with potential users of the VSA technology. The authors aim at the definition of a standardized instrument to spread VSA technology outside the narrow community of its developers, and facilitate access to designing new robots and applications using variable stiffness actuators. The authors believe that this role can be played by a standardized *datasheet* for VSA, and propose one in this paper. The datasheet is intended to be a compact, self-contained description of an actuator that summarizes all its characteristics that are salient to the user, screening him/her from inessential technical implementation details that rather pertain to the VSA designer. The definition of this list of characteristics comes from the authors' experience with both the design of VSA and their integration in variable stiffness robotic systems. A complementary report of such experience gives designer's guidelines for researchers and development engineers willing to face the challenge of designing new VSA systems, and is reported in [Wolf et al., ].

In this paper the list of parameters and figures<sup>1</sup> which compose the datasheet is presented as deriving from the description of the key features (sec. II), control policy (sec. III) and technical requirements (sec. IV) of a VSA. The final datasheet template is presented in sec. V and it is applied as a design instrument in two example applications (sec. VI). The appendix reports an example of the field tests used to derive the parameters contained in the datasheet.

## II. KEY FEATURES OF VSA PERFORMANCE

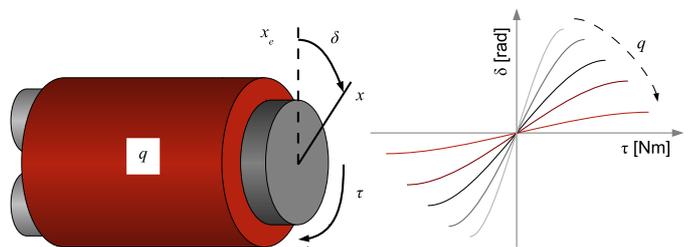


Fig. 2. Base functional schematic of a VSA: an actuator which can move a mechanical load, whose position is indicated by  $x$ , toward an equilibrium position  $x_e$ , allowing a displacement  $\delta = x - x_e$ , determined by the amount of applied torque  $\tau$  and the internal configuration of the actuator  $q$ .

VSAs are, in their very basic essence, mechanical *Actuators*. As such they can be described as systems able to apply torques (or forces) on a mechanical load in order to move it. Moreover, VSAs are flexible actuation systems, thus, (referring to Fig. 2) in consequence of the application of a load  $\tau$  torque to their output shaft, they allow a displacement  $\delta$

<sup>1</sup>For ease of fruition, all the parameters and figures are presented seamlessly throughout the text and are identified by the use of bold fonts. Parameters are also followed by a number between round braces for easy reference.

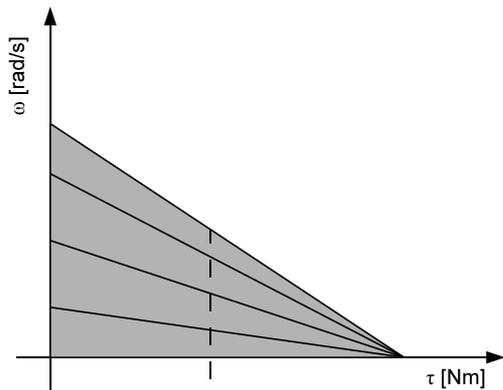


Fig. 3. Speed Vs Torque: A two dimensional chart reporting the output speed (y axis) - output torque (x axis) curve. The dashed line marks the nominal torque of the actuator.

of their output shaft from its equilibrium position  $x_e$ . At rest, this displacement is related to the applied torque by a mechanical stiffness characteristic. The unique feature of a VSA is the ability to dynamically change its mechanical characteristic choosing from a range of possible curves, characterized by different slopes, i.e. by different *stiffness* (assuming stiffness as the slope of the mechanical characteristic is a simplification, feasible here given the scalar nature of the considered mechanical characteristic. For a complete and exhaustive discussion of the concept and usage of stiffness in robotics, please refer to works as [Žefran and Kumar, 2002], [Loncaric, 1985], [Loncaric, 1987], [Howard et al., 1998], [Žefran and Kumar, 1996]).

The first aspect we described puts a VSA on the same level of other motors (e.g. electric motors), from which some parameters are *inherited* directly. In fact, to characterize it, two of the most important aspects are the **Nominal Speed (3)** and **Nominal Torque (2)** the actuator is designed to work at. Together, these two parameters contribute to define the nominal **Continuous Output Power (1)** that the actuator can supply. These parameters, which are among the first numbers a designer looks for when selecting an actuator, are usually limited by thermal considerations. When the actuator is not working continuously, structural strength dictates a less stringent limit **Peak Torque (6)**, and dissipation effects (as friction and CEM forces) determine a **Maximum Speed (7)**. More in general, a quasi-static analysis allows to derive  $\tau$  Vs  $\omega$  plots, as those shown in **Fig. 3**, in complete analogy with other kind of actuators.

Nevertheless, the Variable Stiffness aspect of VSAs extend this traditional two-dimensional characterization of a motor with the range of achievable values of stiffness. This range is ultimately restricted by the **Maximum Stiffness (8)** and **Minimum Stiffness (9)** values. To better characterize the stiffness range of a VSA, its dependence on the supplied torque has to be considered. Since different VSA realization can imply very different relationships, the evolution of  $\sigma$  Vs  $\tau$  should be plotted as in **Fig. 4**. The full information about the  $\tau$ - $\omega$ - $\sigma$  quasi-static workspace of a VSA is intrinsically three-dimensional, thus plots of **Figs 3** and **4** are only two slices of a volume as that shown in **Fig. 5**. After the introduction of the

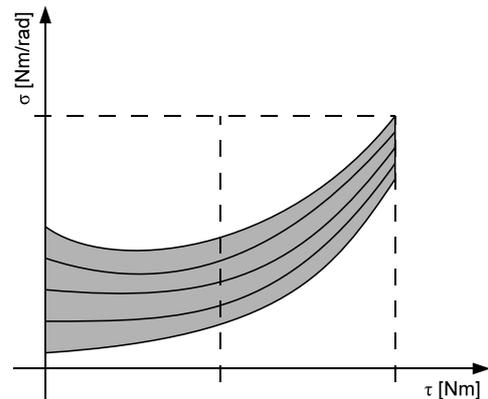


Fig. 4. Stiffness Vs Torque: A two dimensional chart reporting the output stiffness (y axis) - output torque (x axis) curve. The dashed lines mark the nominal torque of the actuator as well as the maximum torque and maximum stiffness.

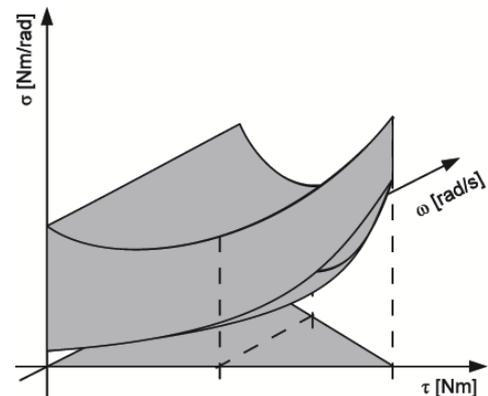


Fig. 5. 3D Workspace: A three dimensional chart reporting the working volume of the actuator in the space defined by output torque (x axis) - output speed (y axis) - output stiffness (z axis). The dashed lines mark the nominal torque of the actuator as well as the maximum torque and maximum stiffness.

nominal stiffness range, we can briefly return on the concepts of **Nominal Speed (3)** and **Nominal Torque (2)**. Since the full characterization of a VSA can be given only considering the three-dimensional torque-speed-stiffness space and because, depending on the VSA implementation, to maintain some level of stiffness can diminish the amount of power left to apply some torque and/or run at some speed, **Nominal Speed (3)** and **Nominal Torque (2)** must take into account this, and their values should be those which can be achieved always within the nominal stiffness range.

Another aspect of a VSA which is really important for some application is the time necessary to change the stiffness from a value to another. In particular since the change of stiffness in a VSA is determined by the movement of a mechanism, change is not instantaneous. The **Nominal Stiffness Variation Time with no load (4)** and **with nominal torque (5)** have to be considered when integrating a VSA in an application where sudden changes of stiffness are important. These two parameters define the time needed to achieve the maximum change of stiffness, with no load and with nominal torque applied to the output shaft, respectively. They are obtained

resorting to the full (peak) torque of the motors internal to the VSA, and must be executed in the worst case scenario (in particular with respect to a change from maximum to minimum stiffness and vice-versa).

The primary application VSAs have been designed for is the operation of robots. By virtue of this, an important property influencing the performance, of the system is the achievable range of motion. In VSAs this range is determined by the contribution of two different values: the **Active Rotation Angle (14)** plus the elastic deflection angle. The maximum elastic deformation, in particular, can vary between a **Maximum deflection with maximum stiffness (12)** and a **Maximum deflection with minimum stiffness (13)**.

Many VSA applications rely on the elasticity of the actuator to store energy. The **Maximum Elastic Energy (10)** the actuator can store in the spring can be important to absorb impacts, to exploit periodic or non-periodic oscillations, and for optimization of energetic behavior.

Lastly, applications which rely on the accuracy of the mechanical characteristic of the actuator for precision, accuracy or repeatability, need to consider the **Maximum Torque Hysteresis (11)** of the actuator, extracted by experimental torque-deflection measurements, as those of **Fig. 8(b)**.

### III. CONTROLLING VSAS

In first approximation, a robot using a VSA is a flexible joint robot system. Literature presented model paradigms (see [Spong, 1987]), and control techniques (as [Albu-Schäffer et al., 2007]) whose ability to tackle the problem of a flexible joint robot, characterized by fixed values of compliance, is established.

Modeling of a flexible joint manipulator with  $n$  links, leads to a  $4n$  state variable system, where the state variables are:

- 1) the  $n$  joint angles or displacements and  $n$  respective speeds
- 2) the  $n$  motor angles and their speeds.

To comply with the larger number of states, a feasible control method consists in using a high gain controller to compensate for the intrinsic joint compliance and be able to use the elastic forces as driving torques of the robot.

Control of a VSA-powered robot could, at least in principle, aim for a similar technique, but this would imply losing most of the benefits derived by the physical variability of compliance by trying to “compensate” for it. Moreover, a joint actuated by a VSA is characterized by a state with 2 more dimensions, where to complete the base to describe all the state, a possible and sound choice is constituted by the stiffness and its rate of change.

The need to comply with this higher complexity and to fully exploit the possibilities offered by Variable Stiffness, is what renders the problem of controlling a VSA non-trivial.

Over the years, a spectrum of different control approaches has been proposed to tackle the problem of VSA control, from the simplest PD control [Tonietti et al., 2005], to feedback linearization [Palli et al., 2008], active damping injection [Petit and Albu-Schaffer, 2011], Immersion and Invariance theory [Wimboeck et al., 2010], and various

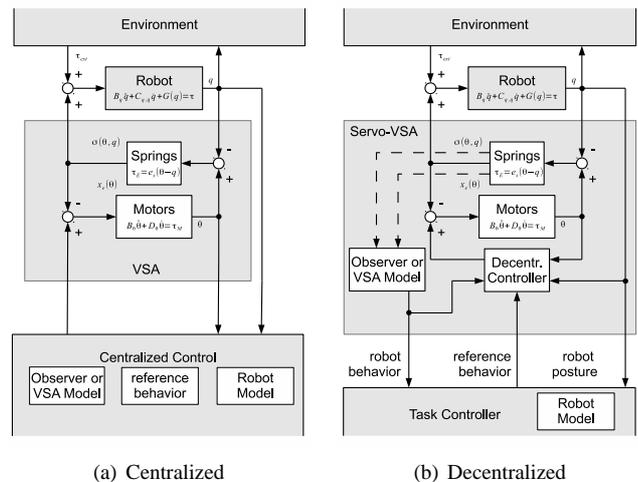


Fig. 6. Two different approaches to the control of a VSA-powered robot. A Centralized controller (left) includes a detailed model of each actuator and manages all the sensory information to optimize the system integration. In the decentralized approach (right) each actuator is locally regulated by dedicated hardware which also interfaces with a simpler central control unit.

flavors of Optimal Control (e.g [Haddadin et al., 2011], [Garabini et al., 2011], [Bicchi et al., 2005] and [Visser et al., 2010]). To bring some order among all these algorithms, they can be sorted based on two viewpoints: On one side, controllers are ordered from centralized to distributed (the two extremes of this distinction are shown in Fig. 6), while on the other side, a different classification of control approaches spans from model-based toward sensor-based.

Different control policies require different knowledge of the VSA system details. The motivation behind the introduction of a VSA in the system usually leads the choice of the most suited control scheme. Indeed, if higher levels of integration and optimization are required, the user can benefit from open systems, where all the knowledge about the system internal model and sensory data is made available. This, in order to be able to define His/Her own centralized and strongly model-optimized control. On the other hand, a component-wise approach could relieve the user from managing all the internal details of the device and let Him/Her concentrate just on the functional aspects. In this case, decentralized hardware should manage the lowest levels of control in place of the user, to let him/her concentrate on the higher layers of the control stack.

In practical terms, the control interface of VSA can accept input commands as simple as equilibrium point and stiffness, in a philosophically similar approach to Equilibrium-Point hypothesis of human motor control (see [Feldman, 1986], [Flanagan and Wing, 1993], [Won and Hogan, 1995]. This kind of black-box system could characterize their behavior with parameters as simple as the **Angular Resolution (15)** of the output shaft sensor.

The availability of further sensory data (e.g. output torque sensors) or the access to the commanded motor torques, allows for more sophisticated control policies even in the case of black-box systems (as suggested by works as

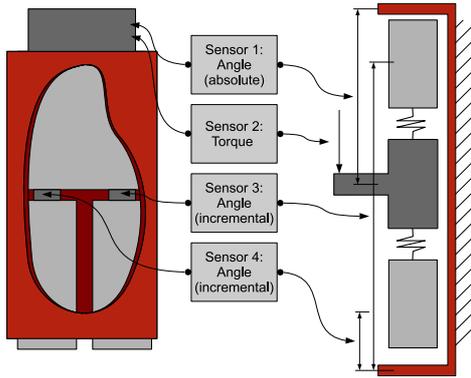


Fig. 7. Sensor map: A logical scheme, with a sufficient detail level, showing the position and purpose of additional sensors inside the actuator.

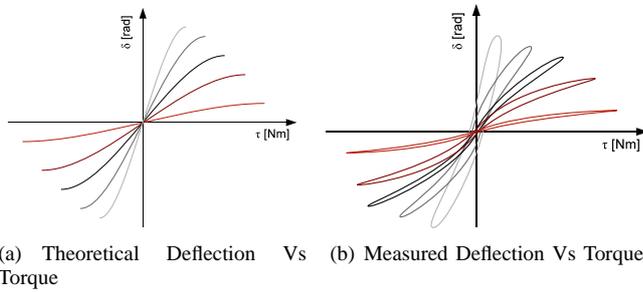


Fig. 8. Charts reporting the deflection (y axis) - torque (x axis) curves, for different values of stiffness preset. Left chart show the trend of the theoretical model, after proper calibration with field data. Right chart reports averaged field (measured) data cycles of loading-unloading, highlighting potential hysteresis phenomena.

[Grioli and Bicchi, 2010], [Flacco and De Luca, 2011]). The only model knowledge of this kind of black-box control requires just a description of the **Recoil Point Function (101)**, which relates the angles of the motors inside the VSA to the equilibrium position of the unloaded output shaft.

More in general, a sensor map, as shown in **Fig. 7**, is fundamental to understand which information is available to be used for feedback control. Each sensor will then be characterized by its table of salient data, as **Resolution (a1)**, **Range (a2)**, **I/O protocol (a3)**, and other implementation dependant properties.

To avoid the adoption of stiffness observers, knowledge of the **Output Torque Function (103)** and, in particular, its derivative the **Output Stiffness Function (104)** is necessary. The former of these functions expresses the torque of the output shaft as functions of the positions of the motors and of the output shaft, the latter gives the values of its derivative w.r.t. the output shaft position, that is, the output stiffness. In particular, for a very fine control, experimental measurements, as in **Figs 8(a)** and **4**, should be taken into account, also compensating for **Maximum Hysteresis (11)**, or for the local one, derived by loading-unloading cycles as those of **Fig.8(b)**.

Most of optimal control approaches, especially when energy expense has to be minimized or power throughput maximized, require full knowledge of the intrinsic system nonlinearities: from the **Energy Function (102)** and the **Spring Torque Function (105)**, relating these quantities to the positions of

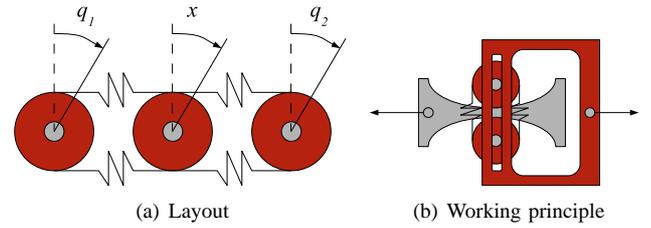


Fig. 9. Actuator Internals: Layout (a) and Working principle (b): Schematic drawing explaining internal layout of the actuator, representing interconnections among the components (motors, elastic transmission, output shaft) and the working principle of the elastic transmission.

the motors and of the output shaft, and finishing with the **Springs to Motors Transmission Ratio (106)** and **Springs to Output Transmission Ratio (107)**. These description are derived from the internal layout and working principles, for the well understanding of which usually a figure as **Fig. 9** is needed.

#### IV. OTHER CHARACTERISTICS OF VSAS

A VSA can be presented to the user from different perspectives: on one extreme, the user gets a non-linear mechanical transmission able to transfer the mechanical work of (usually) two motor sources to one output load though a complex elastic and non-linear coupling system. On the other extreme, the user can get a “black box” with some power and signal inlets and an output shaft, which uses the incoming power and signal to operate the output with different mechanical characteristics.

In both cases, a precise definition of the interfaces separating the VSA from the rest of the system has to make the integration of VSA possible.

Different VSA can use different sources of power to generate mechanical power, the vast literature on the topic introduced not only electro-magnetic devices (which are becoming more and more the standard today), but also pneumatic devices (as cite [Klute et al., 1999], [Verrelst et al., 2006], [Verrelst et al., 2005]), or characterized by novel electric motor technologies as piezoelectric [Clark, 1999], elastomeric [Kornbluh and Pelrine, 2005], active hydrogel [Santulli et al., 2005] and shape memory alloys [Siler and Demoret, 1996] just to name some. Depending on the technology adopted by the device designers, different requirements in terms of power sources would be necessary for the adoption of the VSA in a system. Given the majority of electric devices, we report for example their typical requirements, which can be given in terms of **Nominal Voltage (17)** to drive the device and **Nominal Current (18)** and **Maximum Current (19)** absorbed during the device operation.

Dual to the power interface, a signal interface part would have its specifications, usually expressed in terms of **Voltage Supply (20)** and **Nominal Current (21)** and, above all, the communication **I/O protocol (22)** used to interface with control electronics. All this information needs to be integrated with drawings of the physical connection interface, as in **Fig. 10**. Drawings are important for not only the wiring interface, but the mechanics as well. While a gross approach to the description of a VSA can limit to the overall unit

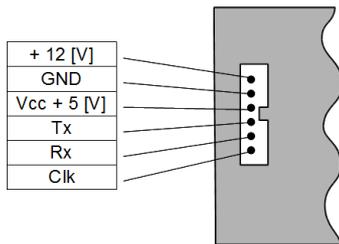


Fig. 10. Connection diagram: The logical scheme showing electronic connections between actuator and external world.

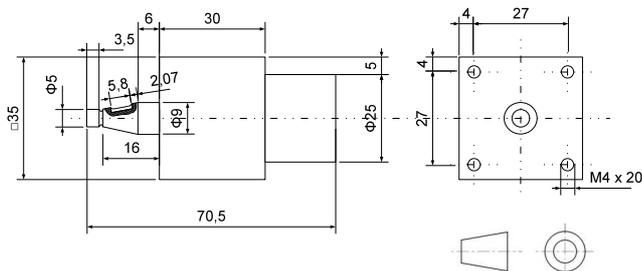


Fig. 11. Mechanical interface drawings: Minimum number of views of the actuator with dimensions defining its size and physical interfaces. Views should comply to European drawing conventions.

**Weight (16)**, more accurate drawings, as those of **Fig. 11**, which highlight the mechanical connection interfaces and the main volumes of the actuator, are the base of good system integration. In the case that the VSA is presented as just a mechanical transmission rather than an integrated units, mechanical drawings would include also connection interfaces of the input shafts.

## V. A VSA DATASHEET

In the light of the former discussions, all of the important parameters have been gathered in the compact form shown in **Figs. 12, 13 and 14**. The main objective pursued in designing it was to obtain a clear and simple document, as the one would expect to find in a catalog.

The datasheet consists of three pages each of which contains a relatively independent set of parameters, representing the system at a different level, with incremental detail.

The first page reports overall characteristics from both the mechanical and electrical point of view. The second page presents some deeper detail about internal sensors, electronics and mechanical performance. Finally in the third page a simple yet complete mechanical scheme of the actuator is reported, as long as a mathematical model of the dynamics of the actuator, expressed in a port-Hamiltonian formalism [Duindam et al., 2009].

For several examples of VSA datasheets and an editable version of the datasheet template see Extensions 1 - 11.

In the Extension VIII several examples of VSA datasheet and an editable version of the datasheet template are available. The material can also be downloaded on the website <http://www.naturalmachinemotion.com>.

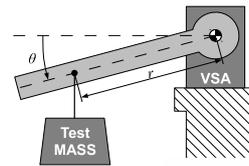


Fig. 15. Schematic representation of the mechanism that enables the measure of torque and deformation for the characterization of a VSA.

## VI. TESTING EXAMPLE

This section illustrates a set of experiments that permits the measurement of salient physical quantities for the characterization of any VSA and hence the compilation of the datasheet template. Procedures to obtain similar data are many and diverse, one conceived for servomotors is reported e.g. in [Tira-Thompson, 2009]. Unfortunately, the authors are not able, at the present time, to suggest universal procedures that can be used to analyse any possible VSA. This section proposes what the authors found most suitable to test the device considered. This procedure can probably be adopted to test also other devices, but it is not inconceivable that a device exists (or will exist in the future) that can not be tested with the presented procedure. Anyhow what is reported in this section can be used as a guideline to develop custom testing procedures.

To fully characterize a VSA system three different load conditions (no load, constant load and variable load), and three kind of experiments have been taken into account. Plots obtained from collected data constitute some of the main design instruments and describe some of the main features of a VSA. The experimental setup consists of the actuator itself and a structure used to apply different loads, along with some off-the-shelf electronic data acquisition board. Force sensors could be used to obtain a better characterization of the system, however it is possible to do the measuring only using the position sensor already inside the actuator (on the prime movers and the output shaft).

### A. Quasi-static load-unload cycles with fixed stiffness preset

The experiments which allow the measure of all the physical quantities needed to fill the datasheet template are presented hereafter. In this experiment, a known torque profile is applied to the actuator, while measuring the position of the output and the motor shafts. This is repeated for different torque values in the feasible range. In this way, torque and deflection profiles, can be measured, therefore, via numerical differentiation, stiffness can be calculated. A possible method to apply a known torque to an actuator is to load it with a known mass  $M$  mounted at a known distance  $r$  from the rotation axis. Exploiting gravity, a varying torque is obtained on the output shaft by rotation, as in Fig. 15. Using symbols of Fig. 15, the torque applied on the actuator is a function of the angular position of the output as

$$\tau = Mgr \cos(\theta). \quad (1)$$

To explore the whole range of feasible torques, from zero to the maximum ( $Mgr$ ), the actuator should be commanded

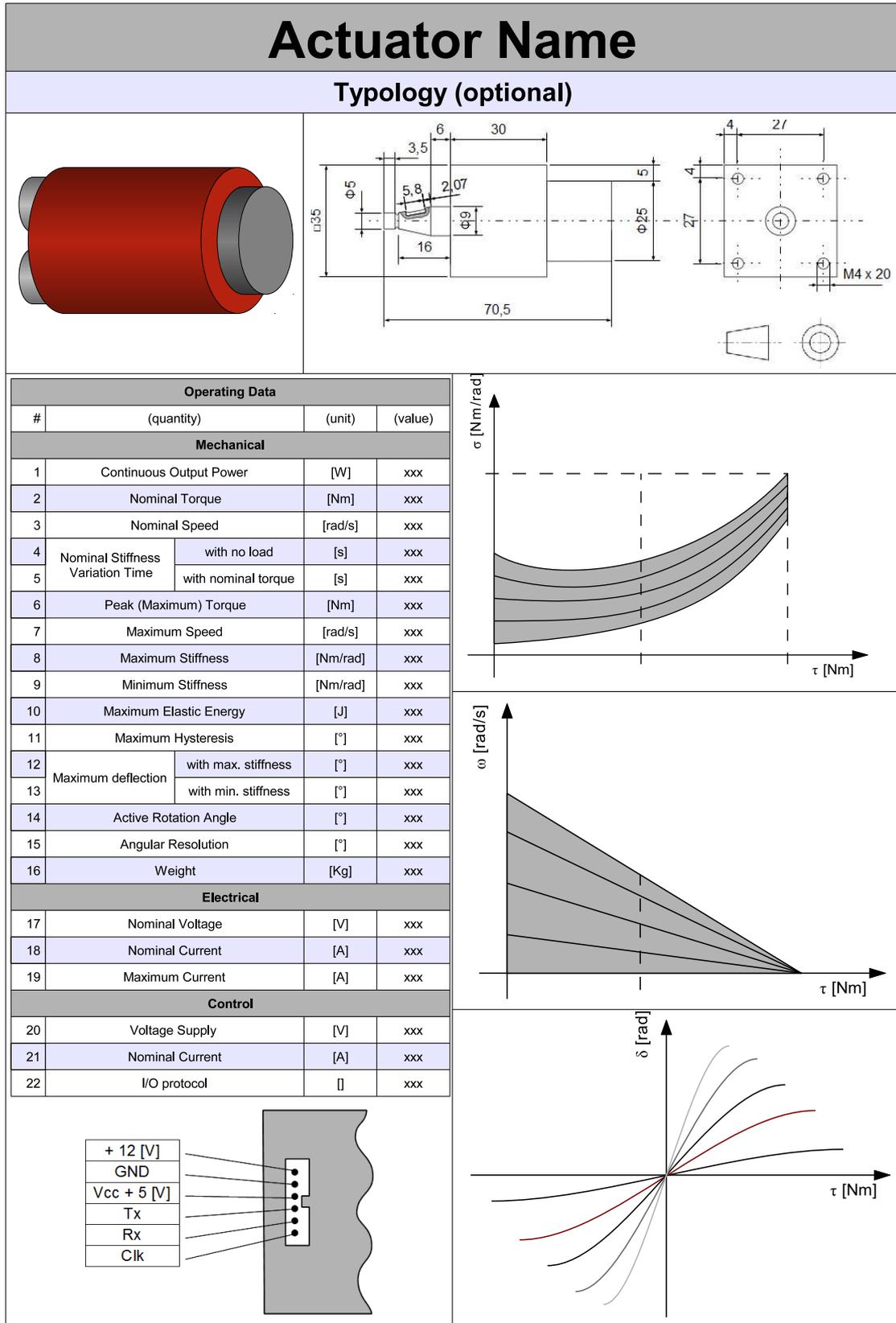


Fig. 12. First page of the VSA datasheet template as it was designed by the VIATORS consortium. It reports overall characteristics from both mechanical and electrical point of view. This page is, in our opinion, not far from the kind of “black-box” datasheet that a “customer” looking for a black-box system, would expect to receive from a “vendor”. See Extensions 10-11 for the datasheet template.

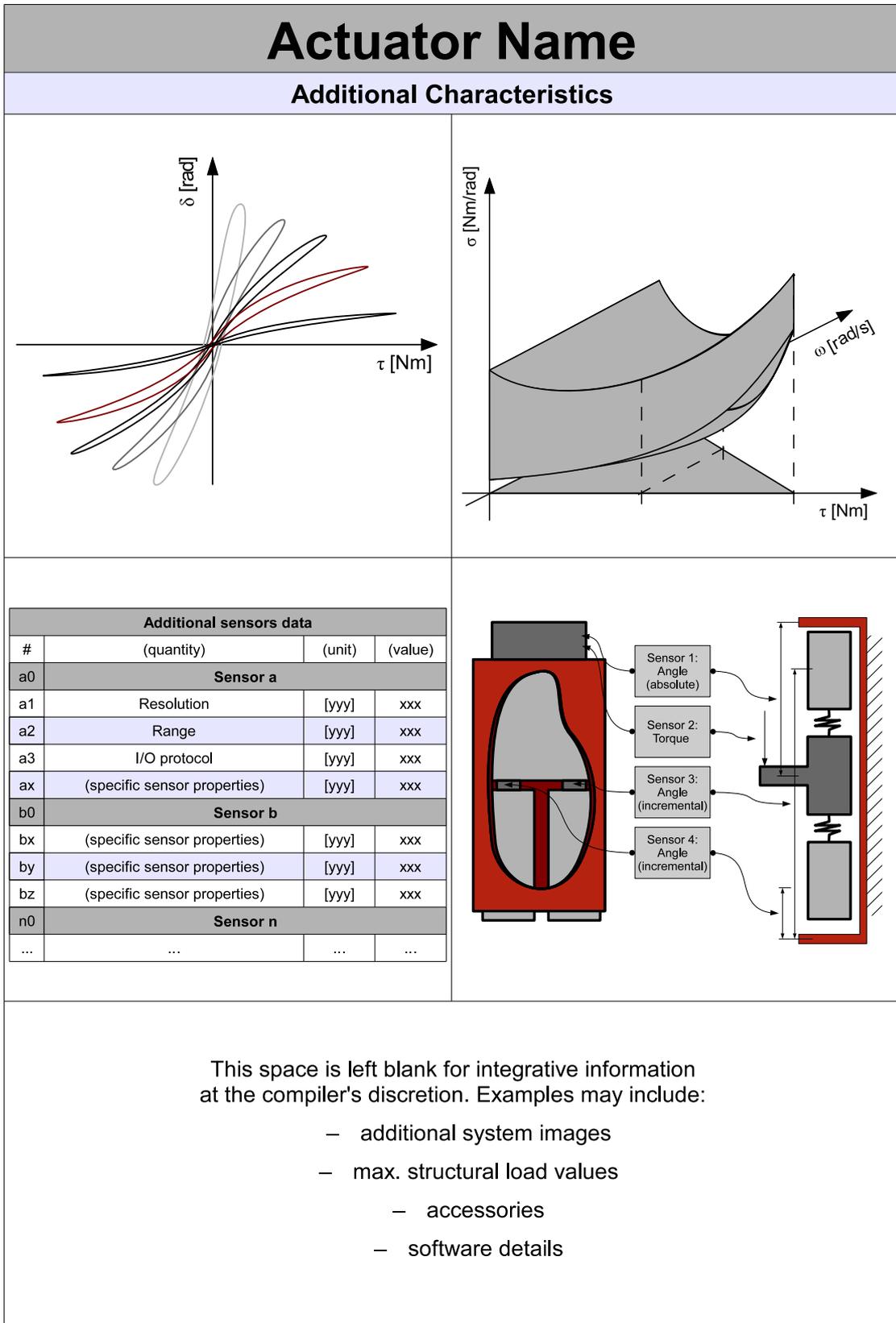


Fig. 13. Second page of the VSA datasheet template as it was designed by the VIATORS consortium. It contains some deeper detail about internal sensors, electronics and mechanical performance are shown. See Extensions 10-11 for the datasheet template.

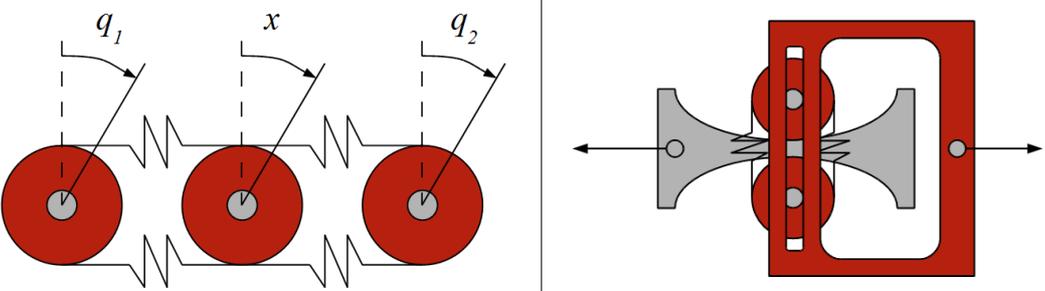
Actuator Name		
Model		
		
Mathematical model		
101	Recoil Point Function	$x_e = x_e(q_1, q_2)$
102	Energy Function	$H = H(q_1, q_2, x)$
103	Output Torque Function	$\tau = \tau(q_1, q_2, x)$
104	Output Stiffness Function	$\sigma = \sigma(q_1, q_2, x)$
105	Spring Torque Function	$e_s = e_s(q_1, q_2, x)$
106	Springs to Motors Transmission Ratio	$A = A(q_1, q_2, x)$
107	Springs to Output Transmission Ratio	$B = B(q_1, q_2, x)$

Fig. 14. Third page of the VSA datasheet template as it was designed by the VIATORS consortium. This page reports a schematic drawing, showing the mechanical functioning of the actuator along with a mathematical model of the actuator dynamics, expressed in port Hamiltonian formalism. See Extensions 10-11 for the datasheet template.

such that the link sweeps from the horizontal position ( $\theta = 0$ ) through the vertical position ( $\theta = \frac{\pi}{2}$ ) and ultimately to the symmetric horizontal ( $\theta = \pi$ ). The sweep has to be executed in quasi static conditions of negligible speed. For instance in the presented tests, a conveniently slow sinusoidal signal reference was used. The test mass used in the example has a weight of  $20\text{ kg}$  with the center of mass at  $0.5\text{ m}$  distance from the center of rotation, corresponding to a maximum torque of about  $100\text{ Nm}$ . Those trials have always been executed with 5 different stiffness presets. The readings from the (three) position sensors have been conveniently filtered.

1) *Torque-deflection and stiffness characteristics*: The obtained torque - deflection curves are plotted in Fig. 16 with varying values of stiffness preset expressed in percentage of the maximum achievable.

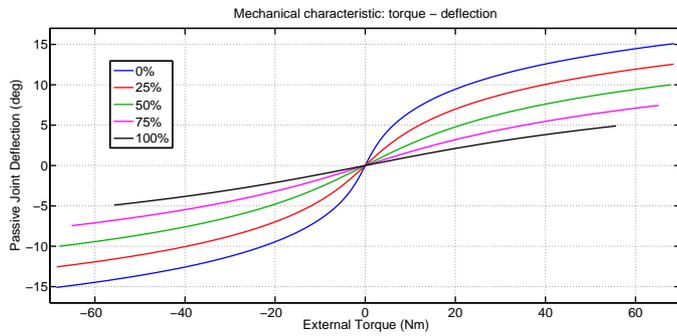


Fig. 16. Torque Vs Deflection: A set of two dimensional curves for different stiffness preset. It reports the output torque on y axis and the output deflection on x axis.

The parameters that can be evaluated from these graphs are:

- **Maximum deflection with maximum stiffness (12)** =  $8.6^\circ$
- **Maximum deflection with minimum stiffness (13)** =  $15.8^\circ$ .

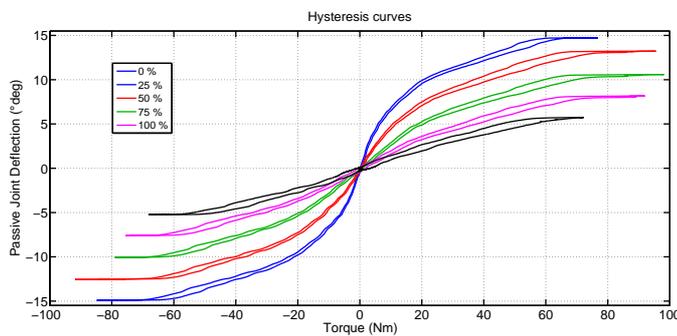


Fig. 17. Hysteresis in the Torque - Deflection plane of the FSJ transmission.

System hysteresis can be evaluated from Fig. 17, from which we obtain **Maximum hysteresis (11)** =  $3^\circ$  Stiffness, obtained via numerical differentiation of torque with respect to deflection, is shown in Fig. 18, obtaining:

- **Maximum stiffness (8)** =  $826 \frac{\text{Nm}}{\text{rad}}$
- **Minimum stiffness (9)** =  $52.4 \frac{\text{Nm}}{\text{rad}}$

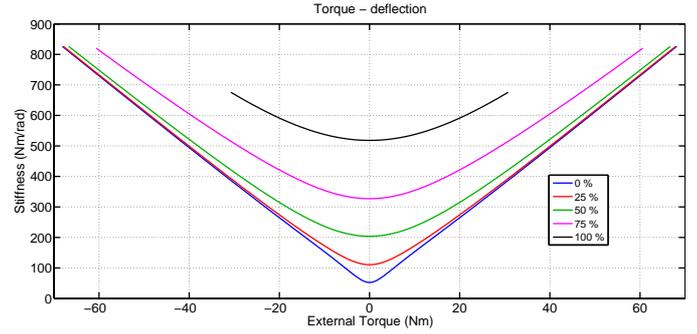


Fig. 18. Stiffness - Torque curves of the transmission of the FSJ plotted for several presets.

## B. Step Command

For the characterization of the maximum speed a step input has been used. The structure described above is not needed in these cases, because no load is required, except for the characterization of stiffness variation time with nominal torque.

*Step Response*: Since the tested device does not provide continuous rotation, the **Maximum Speed (7)** cannot be derived by measuring the output speed in quasi-static rotation regime, hence it is derived from a step response experiment, where the output is commanded the maximum possible step, from one limit of the motion range to the other. Recorded data from the motor angular sensors is averaged over trials, filtered and numerical derivation leads to speed profile reconstruction, from which the **Maximum Speed (7)** is extracted. The experiment is executed for several stiffness presets, as shown in Fig. 19.

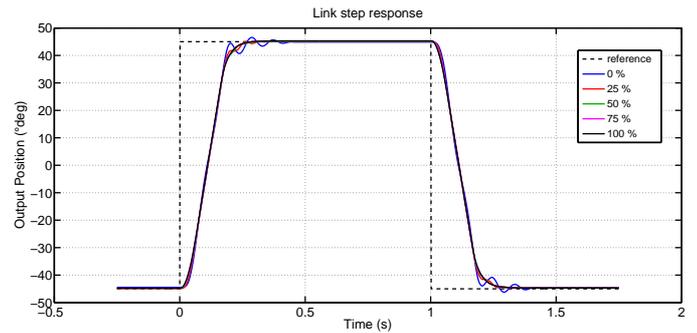


Fig. 19. Response of the output shaft to a step on the equilibrium position input of the transmission of the FSJ, without external load, for different stiffness presets. Maximum speed corresponds to the maximum slope of the responses.

Stiffness variation time with no load is evaluated from the step response to the maximum preset variation, as shown in Fig. 20. **Stiffness variation time with no load (4)** has a value of about  $0.33\text{ s}$ .

Stiffness variation time with nominal torque is obtained with the same procedure as in the no load case with the difference that the nominal torque is applied to the output shaft. The step input and the response of the system is presented in Fig. 20. The value of the **stiffness variation time with nominal torque (5)** is about  $0.33\text{ s}$ .

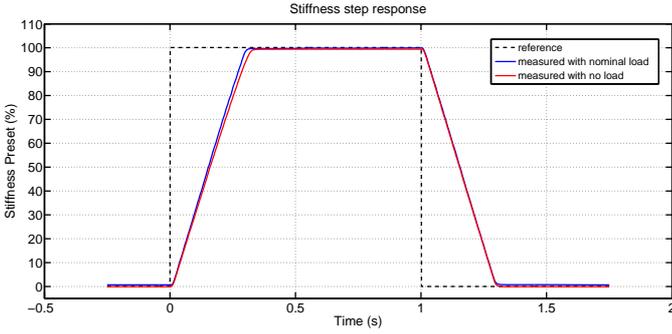


Fig. 20. Response of the measured stiffness preset to inputs signals which generate a step variation of the preset reference in both directions, measured with no load and with nominal load.

## VII. DATASHEET USAGE EXAMPLES

This section guides a potential VSA user through some exemplified procedures for the definition of the main specifications identifying a task in terms of VSA performance parameters. Three different sets of specifications are presented and shortly motivated. The first example uses a VSA for an oscillating cut job. The second example investigates a task of nail hammering.

These examples are introduced as a realistic scenarios nevertheless, for sake of simplicity, one-dimensional models are considered for the translation of the task requirements into VSA specifications. Some considerations about the advantages of VSA over standard stiff actuation are also considered. The two examples are extremely simple but more complex examples would require much deeper analysis that would overflow the scope of this paper. We aim at presenting the scientific community with design examples concerning more complex devices in future publications.

### A. Design example 1: a multi-material cutting tool head.

As a first example consider the problem of cutting using the oscillating movement of a tool, for example a saw or a blade. Depending on the physical and geometrical characteristics of the material being cut, appropriate tools will have different tooth geometry, will be made of different materials and, in general, will present different lengths and weights. Other parameters changing with the particular cut can be swinging speed and oscillation amplitude. A VSA tool-head, equipped with a tool switching mechanism, can easily accommodate for all this kind of changes, efficiently adapting to a range of different operating conditions.

This task is represented via the following data:

- force to cut the material  $F$ ,
- stroke of the cutting blade  $A$ ,
- maximum and minimum inertia of the tool-head  $I_{max}$  and  $I_{min}$ ,
- frequency requirement for the tool movement  $\omega$ .

Suppose that a VSA is used in its linear elasticity range to move the cutting tool head, as depicted in Fig. 21(a). the system can be described via the dynamics of a linear oscillator

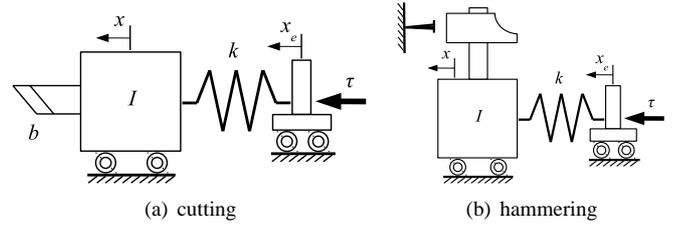


Fig. 21. Translational equivalent of a VSA mounting a cutting tool (left) and a hammer (right).

$$I\ddot{x} + b\dot{x} + k(x - x_e) = 0 \quad (2)$$

where  $x$  represents the displacement of the tool,  $k$  is the variable stiffness of the actuator,  $b$  is the damping value that takes into account the energetic losses during each cutting cycle and  $x_e$  is the displacement of the reference position. The actuator output shaft torque is  $\tau = -k(x - x_e)$ .

Let us assume that the desired tool motion is a sine wave

$$\bar{x} = A \sin(\omega t). \quad (3)$$

Given that the force needed during the cut is a constant with value  $F$ , an equivalent value of  $b$  of (2) can be determined assuming that the energy lost during a cut cycle is equal to the work  $2FA$  done by the cutting force during a cycle, as in

$$\int_0^{2\pi/\omega} b\dot{x}^2 dt = bA^2\omega^2 \frac{\pi}{\omega} = 2FA,$$

where  $\dot{x}$  is the time derivative of the (3). Hence it follows

$$b = \frac{2F}{\pi A\omega}. \quad (4)$$

We will see now how to find the main VSA specifications for our task, that are stiffness, torque and speed ranges.

- **Torque.** The torque range over which the actuator needs to operate, can be determined substituting (3) and its derivatives in (2)

$$\tau = \sqrt{(IA\omega^2)^2 + (bA\omega)^2} \sin(\omega t + \phi_1) \quad (5)$$

where  $\phi_1 = \arctan\left(\frac{I\omega}{b}\right)$ . Considering  $\max_t(\sin(\omega t + \phi_1)) = 1$  and substituting (4), maximum and minimum torques are found as:

$$\tau_{max} = \sqrt{(I_{max}A\omega^2)^2 + \left(\frac{2F}{\pi}\right)^2}, \quad (6)$$

$$\tau_{min} = 0. \quad (7)$$

- **Speed.** The speed range can, once again, be determined by substituting (3) and its derivatives in (2), as

$$\dot{x}_e = \sqrt{\left[A\omega\left(1 - \frac{I\omega^2}{k}\right)\right]^2 + \left(\frac{b}{k}A\omega^2\right)^2} \sin(\omega t + \phi_2) \quad (8)$$

where  $\phi_2 = \arctan\left(\frac{A\omega\left(1 - \frac{I\omega^2}{k}\right)}{\left(\frac{b}{k}A\omega^2\right)}\right)$ .

Under the hypothesis that stiffness is varied only when changing task, to minimize the energy consumption the

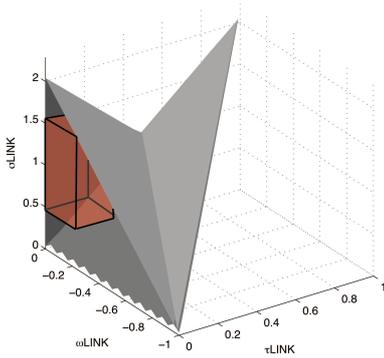


Fig. 22. Fitting the volume demanded by the application in the working volume of the actuator.

speed  $\dot{x}_e$  of the motor has to be minimized. It can be easily shown that the stiffness that minimizes  $\dot{x}_e$  is given by

$$k = \frac{b^2 + I^2\omega^2}{I}. \quad (9)$$

Substituting (9) in (8) and considering  $\max_t(\sin(\omega t + \phi_2)) = 1$  we have a motor speed range of:

$$\dot{x}_{e,max} = \sqrt{\frac{A^2 b^2 \omega^2}{b^2 + m^2 \omega^2}}. \quad (10)$$

While the minimum speed is

$$\dot{x}_{e,min} = 0 \quad (11)$$

- **Stiffness.** From (9) it is possible to value the optimal stiffness to execute a given oscillatory motion. The particular stiffness range would then be a function of the ranges of  $A$ ,  $b$ ,  $\omega$  and  $m$  which, in turn, are all determined by the range of cutting tasks.

Fig. 22 shows an example of fitting of the obtained task specifications in the performance envelope of an actuator. The red volume represents the set of points in the  $\tau - \omega - \sigma$  space that must be included in the set of all working points of a VSA, represented in gray in Fig. 22.

As a comparison, consider the torque and speed ranges required from a traditional (rigid) motor to perform the same task: Torque is the same as for the soft actuator and speed is given by the derivative of (3). The ratio between the speed of soft and rigid is

$$r_\omega = \frac{b}{\sqrt{b^2 + I^2\omega^2}} \quad (12)$$

Substituting the equivalent damping this becomes

$$r_\omega = \frac{2F}{\sqrt{4F^2 + \pi^2 I^2 A^2 \omega^4}}. \quad (13)$$

This ratio is lower than one. Hence, a properly designed soft actuator requires a slower motor than a rigid one with same nominal torque. Of course this advantage becomes more pronounced the more the term  $\pi^2 I^2 A^2 \omega^4$  is large when compared to  $4F^2$ . An example of the trend of  $r_\omega$  can be seen in Fig. 23. By the way, in such a device, the choice of  $K$  could be done adaptively on line by minimizing the

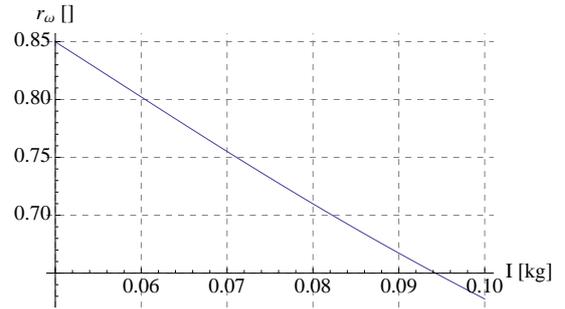


Fig. 23. Trend of  $r_\omega$ , ratio between the maximum required speeds of a soft actuator with respect to a rigid actuator, as a function of the inertia of the cutting tool. The other parameters are  $A = 10mm$ ,  $\omega = 50Hz$ ,  $F = 125N$ .

energy consumption, for sake of simplicity the procedure to implement such functionality is not discussed here.

#### B. Design example 2: A multipurpose tool-head.

Suppose that a user wants to employ the highly-dynamic behaviors of the VSA tool-head to move a hammer-like tool for driving nails, like in the scheme of Fig. 21(b). As shown in works such as [Garabini et al., 2011], [Haddadin et al., 2008], [Haddadin et al., 2011] and [Hondo and Mizuuchi, 2011], this task can be formalized as an optimal control problem in which the kinetic energy of the link attached to the actuator, in this case the hammer tool, is maximized at the instant of the impact. The solution of this problem, which is sensitive to boundary conditions, such as maximum limits on the link motion range and the eventual time specified for the impact, is not always trivial. Nevertheless, if optimality of the solution is not a stringent request and link and motor motion ranges are sufficiently large, a preliminary dimensioning of the actuator can be found thanks to the energy conservation principle.

For this preliminary actuator selection, we can assume the task can be successfully accomplished if, at the very moment of the impact, the link possesses a sufficient amount of kinetic energy  $T_{impact}$ . If a VSA is used to sway a link of inertia  $I$ , the total mechanical energy  $L$  in the system comprising the link and the elastic transmission is

$$L = U_e + T. \quad (14)$$

In (14),  $U_e$  is the elastic potential energy in the variable stiffness transmission, and  $T$  the kinetic energy in the link. A likely control policy for hammering consists in loading the actuator with the maximum mechanical energy  $L_{max}$  the actuator can provide within nominal working conditions, and then rapidly discharge all of it in the kinetic component  $T_{impact}$ , to maximize the drive on the nail.

Both terms that sum up to  $L$  are limited due to physical limits in the actuator, in particular

$$U_e \leq U_{max} \quad (15)$$

where  $U_{max}$  is the **Maximum Elastic Energy (10)**, while the kinetic energy the actuator can confer to the link in steady

state is limited by the **Maximum Speed (7)**,  $\omega_{max}$ , through

$$T \leq T_{max} = \frac{1}{2} I \dot{\omega}_{max}^2. \quad (16)$$

Assuming that all the elastic energy gets converted in kinetic energy without losses, the maximum kinetic energy of the link yields

$$T_{impact} \approx L_{max} = U_{max} + T_{max}, \quad (17)$$

thus, concerning the maximum speed of the tool-head, it yields

$$\dot{x}_{max} = \sqrt{\dot{\omega}_{max}^2 + \frac{2U_{max}}{I}}. \quad (18)$$

If, in practice, just a proportion  $p$  of the **Maximum Speed (7)** is considered, and coefficient of restitution  $\mu$  is considered, (18) becomes

$$\dot{x}_{max} = \sqrt{\mu} \sqrt{p^2 \omega_{max}^2 + \frac{2U_{max}}{I}}. \quad (19)$$

This preliminary analysis shows how, given a target hammer speed, it can be reached by a VSA with a smaller nominal maximum speed, exploiting the elastic energy it can store in the transmission. Aware of this, a proper actuator can be chosen as long as it fulfill the limitations of equation (19).

It is important not to forget that to achieve the task in a reasonable amount of time, the **Peak Torque (6)**  $\tau_{max}$  of the actuator has to be taken into account as well. In fact, using a first order approximation, of the actuator Speed Vs Torque characteristic (i.e. **Fig. 3**), and assuming the inertia  $I$  is accelerated up to a fraction  $p$  of the theoretical maximum speed, the time  $t_t$  to accomplish the task can be estimated as

$$t_t = nI \frac{\log(1-p)\omega_{max}}{\tau_{max}}, \quad (20)$$

where  $n$  is a reality factor accounting for other neglected times such as, e.g. the time needed to load the springs. Equation (20) highlights how the time  $t_t$  is in inverse proportion to the **Peak Torque (6)**.

## VIII. CONCLUSIONS

Variable Stiffness Actuation has slowly moved the step which separates conceptual state of the art from proved prototypes. It is now ready to move the next step toward becoming a solid, reliable technology. This paper tries to move away from the habitual viewpoint that usually describes VSA, which is that of their designers. To establish a common ground where to confront with future potential users of VSA technology, a VSA datasheet has been presented as an interface language between designers and users. The three pages resulting template resumes all the key features characterizing the performance, and issues relative to control and interfacing of a VSA as a component of a system. In the end of the paper the datasheet of some existing actuators were presented as an example, as well as two guided design procedures.

Possibly in the future we envision to use the Datasheet as a base tool in the design of novel service robots, for applications such as manufacture and assembly, human-robot interaction and more in general co-working.

## ACKNOWLEDGMENT

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## APPENDIX A: INDEX TO MULTIMEDIA EXTENSIONS

Extension	Media Type	Description
1	Datasheet	Datasheet of the DLR-FSJ
2	Datasheet	Datasheet of the DLR-fas
3	Datasheet	Datasheet of the DLR-BAVS
4	Datasheet	Datasheet of the VSA-CUBE
5	Datasheet	Datasheet of the VSA-HD
6	Datasheet	Datasheet of the AWAS
7	Datasheet	Datasheet of the AWAS-II
8	Datasheet	Datasheet of the vsaUT
9	Datasheet	Datasheet of the MACCEPA
10	Datasheet	Datasheet Template
11	Datasheet	Editable Datasheet Template

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