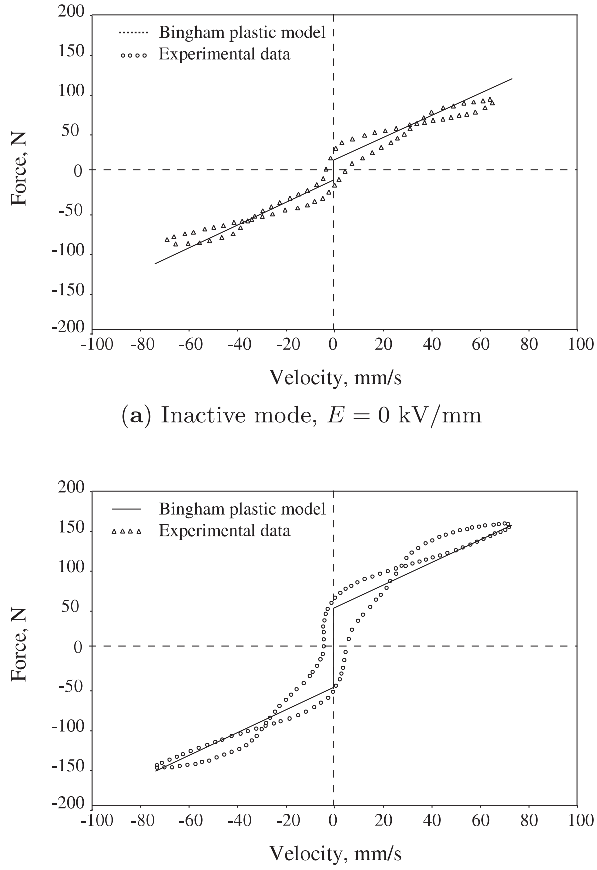


(a) Inactive mode, $E = 0$ kV/mm

(b) Active mode, $E = 3$ kV/mm

Figure 7.21. Comparison of experimental damper data with Bingham plastic model predictions, at electric fields of $E = 0$ kV/mm and $E = 3$ kV/mm, adapted from Kamath et al. [16].

behavior of a damper in the shear mode as well as in the flow mode, using a Bingham plastic fluid model. We first consider the simplest flow geometry, which is a passage of rectangular cross section, and then explore an annular flow passage, which is more suited for many practical engineering applications. The flow passage of rectangular cross section is formed by the gap between two parallel plates that also act as the electrodes (in the case of ER fluids) or magnetic poles (in the case of MR fluids) for the application of a field. An annular flow passage can be formed by the gap between two concentric cylinders that also act as the electrodes or magnetic poles. The behavior of dampers in the squeeze mode is not considered here; discussion of this aspect has been provided by Stanway et al. [47, 48].

7.4.1 Rectangular Flow Passage

Let us consider the behavior of the fluid in a passage of rectangular cross section. The fluid is enclosed between two parallel plates that also form the electrodes or magnetic poles. An electric or magnetic field is applied across the height of the passage d. The length over which the field is applied, or the active length, is L, and the width of the passage is b. A schematic of this flow passage is shown in Fig. 7.22. It can be assumed that a uniform field exists across the height of the passage, over an area $L \times b$. The fluid enclosed in this volume forms a simple active fluid element.

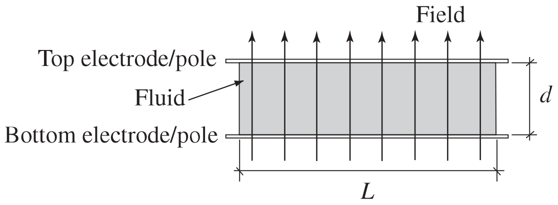


Figure 7.22. ER/MR fluid in rectangular flow passage.

Consider the force equilibrium on a rectangular fluid element of length dx, height dy, and width b as shown in Fig. 7.23. The force equilibrium equation can be written as

$$-m\ddot{x} + P\,dy\,b - \tau\,dx\,b - \left(P + \frac{\partial P}{\partial x}\,dx\right)dy\,b + \left(\tau + \frac{\partial \tau}{\partial y}\,dy\right)dx\,b = 0 \quad (7.19)$$

where P is the fluid pressure, τ is the shear stress, and m is the mass of the fluid element given by

$$m = \rho\,dy\,dx\,b \quad (7.20)$$

where ρ is the mass density of the fluid. Substituting in Eq. 8.171, we obtain

$$-\rho\frac{\partial u}{\partial t} - \frac{\partial P}{\partial x} + \frac{\partial \tau}{\partial y} = 0 \quad (7.21)$$

where u is the axial velocity $(\partial x/\partial t)$. Assuming a quasi-steady flow

$$\frac{\partial u}{\partial t} = 0 \quad (7.22)$$

The governing equation reduces to

$$\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x} \quad (7.23)$$

We examine the behavior of a damper using this active fluid element operating in two modes: shear mode and flow mode.

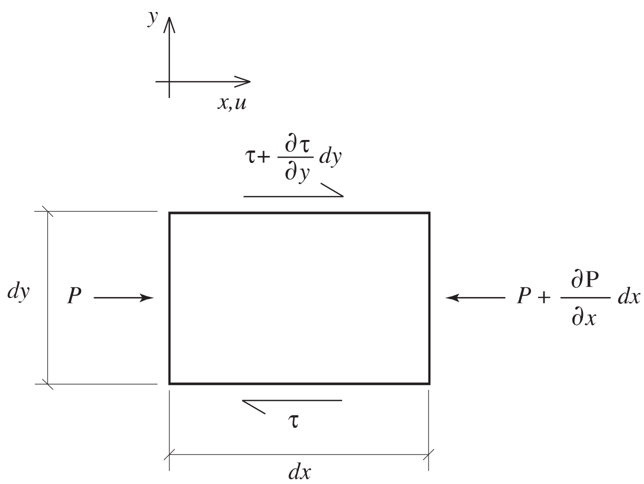
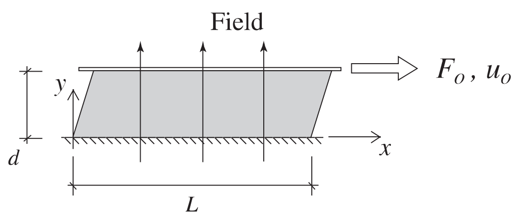


Figure 7.23. Differential fluid element, rectangular flow passage.

Figure 7.24. Rectangular flow passage: shear mode operation.



Shear Mode

A shear mode damper can be constructed with the rectangular flow geometry shown in Fig. 7.22 by moving the upper plate with respect to the lower one, while maintaining a constant gap d between them. Assume that a force F_o acts on the upper plate, moving it with a constant velocity u_o. A schematic of this configuration is shown in Fig. 7.24. In this case, the pressure gradient is

$$\frac{\partial P}{\partial x} = 0 \tag{7.24}$$

The governing equation reduces to

$$\frac{\partial \tau}{\partial y} = 0 \tag{7.25}$$

(a) Solution under zero applied field

When no field is applied, the fluid behaves like a Newtonian fluid. The shear stress is given by (Eq. 7.1)

$$\tau = \mu \frac{\partial u}{\partial y} \tag{7.26}$$

where μ is the dynamic viscosity of the fluid. Substituting in Eq 7.25, we obtain

$$\mu \frac{\partial^2 u}{\partial y^2} = 0 \tag{7.27}$$

Integrating twice leads to

$$u(y) = A\,y + B \tag{7.28}$$

The constants A and B are determined from the boundary conditions

$$\begin{cases} u(0) = 0 \\ u(d) = u_o \end{cases} \quad \Longrightarrow \quad \begin{cases} B = 0 \\ A = u_o/d \end{cases} \tag{7.29}$$

The velocity profile is given by

$$u(y) = \frac{u_o}{d}\, y \tag{7.30}$$

and the shear stress is

$$\tau(y) = \mu \frac{\partial u}{\partial y} = \mu \frac{u_o}{d} \tag{7.31}$$

The force on the upper plate required to move it with the velocity u_o is given by

$$\begin{aligned} F_o &= \tau(d)\, L\, b \\ &= \mu\, \frac{u_o}{d}\, L\, b \end{aligned} \tag{7.32}$$

This can be equated to the equivalent damping force, yielding an effective damping coefficient (inactive state) c^o_{eq}.

$$F_o = c^o_{eq}\, u_o \quad \Longrightarrow \quad c^o_{eq} = \frac{\mu\, L\, b}{d} = \mu\, \Gamma \tag{7.33}$$

where Γ is a parameter that depends only on the geometry of the flow passage.

(b) Solution under non-zero applied field

When a field is applied across the gap, the fluid is modeled as a Bingham plastic. The shear stress is given by

$$\tau(y) = \tau_y + \mu\, \frac{\partial u}{\partial y} \tag{7.34}$$

The velocity profile is calculated from the governing Eq. 7.25. Because τ_y is independent of y, and the boundary conditions are the same, the velocity profile is the same as before

$$u(y) = \frac{u_o}{d}\, y \tag{7.35}$$

The shear stress is given by

$$\tau(y) = \tau_y + \mu\, \frac{u_o}{d} \tag{7.36}$$

and the force in the damper is

$$\begin{aligned} F_o &= \tau(d)\, L\, b \\ &= \left(\tau_y + \mu \frac{u_o}{d}\right)\, L\, b \\ &= \left(\frac{\tau_y\, d}{\mu\, u_o} + 1\right) \mu \frac{u_o}{d}\, L\, b \\ &= c^a_{eq}\, u_o \end{aligned} \tag{7.37}$$

where c^a_{eq} is the effective damping coefficient in the active state, defined as

$$c^a_{eq} = \mu\Gamma(1 + \mathsf{Bi}) \tag{7.38}$$

The quantity Bi is called the Bingham number and is a nondimensional quantity relating the yield stress to the viscous stress. Introducing nondimensional quantities in the analysis, such as the Bingham number and other parameters based on the damper geometry, enables the performance of different types and sizes of devices to be compared on the same basis. Note that if the velocity u_o is high, then the Bingham number is small and, consequently, the increase in damping coefficient on activation of the fluid is small. It can be concluded that when an activated fluid is subjected to high velocities, because the Bingham number is small, the fluid tends to behave more like a Newtonian fluid than like a Bingham plastic. Therefore, the displacement

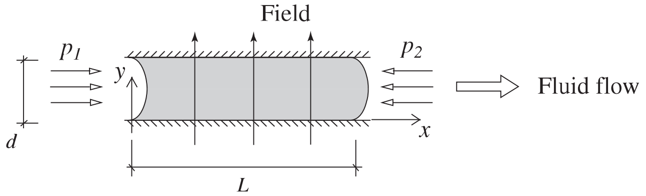


Figure 7.25. Rectangular flow passage: flow mode operation.

amplitude and operating frequency are also important parameters in characterizing the performance of a damper. The expression for the Bingham number is

$$\mathsf{Bi} = \frac{\tau_y}{\mu u_o/d} = \frac{\text{yield stress}}{\text{viscous stress}} \tag{7.39}$$

It can be seen that the Bingham number depends on the yield stress and viscosity of the fluid, as well as on the gap height and the velocity of motion. The smaller the gap, the smaller the Bingham number. Note that for a Newtonian fluid, the Bingham number is zero. The equivalent active damping coefficient, c^a_{eq}, can be written as (from Eq. 7.38)

$$c^a_{eq} = c^o_{eq}(1 + \mathsf{Bi}) \tag{7.40}$$

We see that the damping coefficient in the active state has increased by the amount Bi. Therefore, Bi defines the amount of active damping in the device. To create the largest change in damping on the application of a field, the ratio of active damping coefficient to inactive damping coefficient must be high. Therefore

$$\frac{c^a_{eq}}{c^o_{eq}} \gg 1 \quad \rightarrow \quad \mathsf{Bi} \gg 1 \quad \rightarrow \tau_y \gg \frac{\mu u_o}{d} \tag{7.41}$$

This means that the yield stress must be much higher than the viscous stress. Because u_o is based on the application and d is based on the geometry of the device, the ideal controllable fluid should have a high yield stress τ_y and a low dynamic viscosity μ.

Flow Mode

A flow mode damper can be constructed with the rectangular flow geometry shown in Fig. 7.22 by holding both of the plates fixed and creating a fluid flow between them. A schematic of this configuration is shown in Fig. 7.25. The fluid flow is caused by the difference in pressures p_1 and p_2 at the ends of the flow passage. In this case, the pressure gradient is related to the applied differential pressure ΔP across the active length (assumed constant over the entire active length). Note that $\Delta P = p_1 - p_2$ is the pressure drop across the length of the gap. The pressure gradient is given by

$$\frac{\partial P}{\partial x} = -\frac{\Delta P}{L} = \frac{p_2 - p_1}{L} \tag{7.42}$$

It is assumed that the location under consideration is sufficiently far away from the ends of the flow passage such that the flow profile is fully developed. The governing

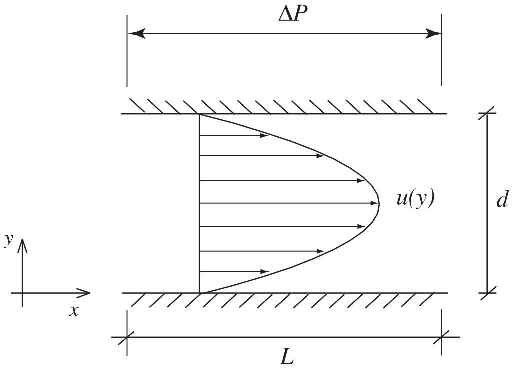


Figure 7.26. Flow profile of the fluid in the flow mode, no field applied (rectangular cross section).

equation becomes

$$\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x} = -\frac{\Delta P}{L} \tag{7.43}$$

(a) Solution under zero applied field

In the inactive state, the fluid behavior is Newtonian. The governing equation becomes

$$\mu \frac{\partial^2 u}{\partial y^2} = -\frac{\Delta P}{L} \tag{7.44}$$

Integrating twice yields

$$u(y) = -\frac{\Delta P}{2\mu L} y^2 + Cy + D \tag{7.45}$$

The constants C and D are determined from the boundary conditions

$$\begin{cases} u(0) = 0 \\ u(d) = 0 \end{cases} \quad \Longrightarrow \quad \begin{cases} D = 0 \\ C = \frac{d}{2}\frac{\Delta P}{\mu L} \end{cases} \tag{7.46}$$

Substituting these constants into Eq. 7.45, the velocity profile of the flow across the gap can be written as

$$\begin{aligned} u(y) &= -\frac{\Delta P}{2\mu L} y^2 + \frac{\Delta P d}{2\mu L} y \\ &= \frac{\Delta P}{2\mu L} y(d - y) \end{aligned} \tag{7.47}$$

It can be seen that the velocity profile is parabolic (shown in Fig. 7.26). By symmetry, it is evident that the velocity is maximum at the center of the gap

$$\begin{aligned} u(d/2) &= u_o \\ &= \frac{\Delta P}{2\mu L} \frac{d}{2} \frac{d}{2} \\ &= \frac{\Delta P d^2}{8\mu L} \end{aligned} \tag{7.48}$$

The velocity profile can also be conveniently expressed in nondimensional form

$$\bar{u}(\bar{y}) = 4\bar{y}(1-\bar{y}) \tag{7.49}$$

where

$$\bar{y} = \frac{y}{d} \quad \text{and} \quad \bar{u} = \frac{u}{u_o} \tag{7.50}$$

The shear stress in the gap is

$$\begin{aligned} \tau(y) &= \mu\frac{\partial u}{\partial y} = \mu\left(-\frac{\Delta P y}{\mu L} + \frac{\Delta P d}{2\mu L}\right) \\ &= \frac{\Delta P}{L}\left(\frac{d}{2} - y\right) \end{aligned} \tag{7.51}$$

The force required to maintain the flow velocity in the passage, which is basically the damping force in the device, is given by the product of the differential pressure and the cross-sectional area. We can assume that the flow is created by a piston with the same cross section as the flow passage, moving with a constant velocity u_o. The force required to move the piston is F_o. Because the velocity profile across the gap is parabolic, a mean velocity u_m can be defined that is constant across the gap and that yields the same volumetric flow as the parabolic profile. The volumetric flow Q is given by

$$\begin{aligned} Q &= \int_{y=0}^{d} u(y) b dy = b\int_0^d \frac{\Delta P}{2\mu L} y(d-y) dy \\ &= \frac{\Delta P b}{2\mu L}\left[\frac{dy^2}{2} - \frac{y^3}{3}\right]_0^d \\ &= \frac{\Delta P b d^3}{12\mu L} \end{aligned} \tag{7.52}$$

The volumetric flow can also be expressed in terms of the mean velocity, u_m, as follows

$$Q = u_m b d \tag{7.53}$$

From Eqs. 7.52 and 7.53, the mean velocity is

$$u_m = \frac{\Delta P d^2}{12\mu L} \tag{7.54}$$

The damping coefficient of the fluid element can be found from the force and velocity of the piston. The differential pressure is related to the force on the piston by

$$\Delta P = \frac{F}{bd} \tag{7.55}$$

which yields

$$F = \frac{12\mu b L}{d} u_m \tag{7.56}$$

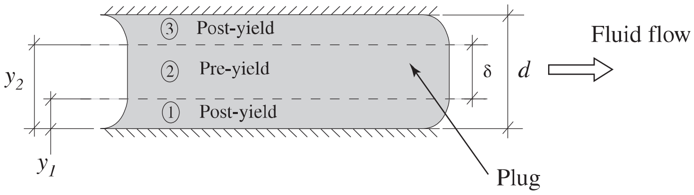


Figure 7.27. Flow profile of the fluid in the flow mode, under an external field (rectangular cross section), Bingham plastic model.

The damping coefficient of the fluid element under zero applied field, c_{eq}^o, can be found from the previous equation as

$$\begin{aligned} c_{eq}^o &= \frac{F}{u_m} \\ &= \frac{12\mu Lb}{d} \end{aligned} \tag{7.57}$$

It is seen that the damping coefficient depends on the geometry of the damper and the viscosity of the fluid.

(b) Solution under non-zero applied field

When a field is applied, the velocity profile of the fluid changes depending on the local shear stress. The flow velocity profile in the Newtonian case is parabolic and the shear stress at the middle of the gap is zero. Therefore, around this region, the fluid is in the pre-yield condition. Near the walls of the passage, the shear stresses may be higher than the yield stress, resulting in post-yield fluid behavior. Treating the fluid as a Bingham plastic, it can be seen that in the pre-yield region, the fluid behaves like a solid and, therefore, has a constant translational velocity around the center of the gap. Near the walls, the fluid behavior is Newtonian, with a parabolic velocity profile. The resulting flow profile across the height of the gap can be considered as a solid plug around the center of the gap, being carried along in a Newtonian fluid. This flow profile is depicted in Fig. 7.27. The flow is divided into three regions: regions 1 and 3 are the post-yield regions and region 2 is the pre-yield region. The thickness of the plug in the center of the gap is δ.

To find the flow profile in the gap and the effective damping coefficient, each of the three regions is treated separately. Substituting the expressions for shear stress in each region, we see that the governing equation for all three regions reduces to Eq. (7.44)

$$\mu \frac{\partial^2 u}{\partial y^2} = -\frac{\Delta P}{L} \tag{7.58}$$

and the location of each region is

$$y_1 = \frac{d - \delta}{2} \tag{7.59}$$

$$y_2 = \frac{d + \delta}{2} \tag{7.60}$$

Region 1

Integrating the previous governing equation twice leads to

$$u_1(y) = -\frac{\Delta P}{2\mu L}y^2 + C_1 y + C_2 \tag{7.61}$$

The boundary conditions in this case are

$$u_1(0) = 0 \tag{7.62}$$

$$\left.\frac{\partial u_1}{\partial y}\right|_{y=y_1} = 0 \tag{7.63}$$

Whereas the first boundary condition is a result of the no-slip condition at the wall, the second boundary condition occurs because there can be no discontinuity in the flow profile. Substituting and solving yields the constants

$$C_2 = 0 \tag{7.64}$$

$$-\frac{\Delta P}{\mu L}y_1 + C_1 = 0 \implies C_1 = \frac{\Delta P y_1}{\mu L} \tag{7.65}$$

Therefore, the velocity profile in region 1 is given by

$$\begin{aligned} u_1(y) &= -\frac{\Delta P}{2\mu L}y^2 + \frac{\Delta P y_1}{\mu L}y \\ &= \frac{\Delta P}{2\mu L}\, y\,(2y_1 - y) \\ &= \frac{\Delta P}{2\mu L}\, y\,(d - \delta - y) \end{aligned} \tag{7.66}$$

Region 3

Integrating the governing equation twice leads to

$$u_3(y) = -\frac{\Delta P}{2\mu L}y^2 + C_3 y + C_4 \tag{7.67}$$

The boundary conditions in this case are

$$u_3(d) = 0 \tag{7.68}$$

$$\left.\frac{\partial u_3}{\partial y}\right|_{y_2} = 0 \tag{7.69}$$

These boundary conditions are similar to that of the previous case. Substituting and solving yields the constants

$$-\frac{\Delta P}{\mu L}y_2 + C_3 = 0 \implies C_3 = \frac{\Delta P y_2}{\mu L} \tag{7.70}$$

$$-\frac{\Delta P}{2\mu L}d^2 + \frac{\Delta P}{\mu L}d\, y_2 + C_4 = 0 \implies C_4 = \frac{\Delta P}{2\mu L}\, d\,(d - 2y_2) \tag{7.71}$$

Therefore, the velocity profile in region 3 is given by

$$\begin{aligned} u_3(y) &= -\frac{\Delta P}{2\mu L} y^2 + \frac{\Delta P}{\mu L} y y_2 + \frac{\Delta P}{2\mu L}\left(d^2 - 2d\, y_2\right) \\ &= \frac{\Delta P}{2\mu L}\left[\left(d^2 - y^2\right) - 2y_2(d-y)\right] \\ &= \frac{\Delta P}{2\mu L}(d-y)(y-\delta) \end{aligned} \tag{7.72}$$

Note that this result can also be obtained from the symmetry of the flow

$$u_3(y) = u_1(d-y) \tag{7.73}$$

Applying this relation to Eq. 7.66 results in Eq. 7.72.

Region 2

The velocity is constant in region 2, given by the velocity at the locations y_1 and y_2. Let us call the velocity of the fluid in region 2 the plug velocity, u_p. Then we can write

$$\begin{aligned} u_1(y_1) &= u_p \\ u_3(y_2) &= u_p \end{aligned} \tag{7.74}$$

Substituting in Eq. 7.66, we obtain

$$\begin{aligned} u_p = u_1(y_1) &= \frac{\Delta P}{2\mu L} y_1^2 \\ &= \frac{\Delta P (d-\delta)^2}{8\mu L} \end{aligned} \tag{7.75}$$

As a check

$$\begin{aligned} u_3(y_2) &= \frac{\Delta P}{2\mu L}(d-y_2)(y_2-\delta) \\ &= \frac{\Delta P}{8\mu L}(d-\delta)^2 \\ &= u_p \end{aligned} \tag{7.76}$$

Note that the solution of the governing flow equation (Eq. 7.43) in all three regions involves a total of five constants: C_1, C_2, C_3, C_4, and δ. The boundary conditions in regions 1 and 3 (Eqs. 7.62, 7.63, 7.68, and 7.69) provide four equations. The condition of equal flow velocities at the locations y_1 and y_2 (Eq. 7.74) does not provide any additional information because y_1 and y_2 are fixed by the assumption that the flow profile is symmetric about the center of the flow passage. Therefore, an additional condition is required to find the thickness of the plug, δ. This can be found by solving for the shear stress at the boundary of region 2. The governing equation (Eq. 7.43) in region 2 is written as

$$\frac{\partial \tau_2}{\partial y} = -\frac{\Delta P}{L} \tag{7.77}$$

Integrating this equation yields

$$\tau_2(y) = -\frac{\Delta P}{L} y + C_5 \tag{7.78}$$

The constants δ and C_5 can be found from the following boundary conditions

$$\tau_2(y_1) = \tau_y \tag{7.79}$$

$$\tau_2(y_2) = -\tau_y \tag{7.80}$$

Substitution in Eq. 7.78 results in an expression for C_5

$$C_5 = \frac{\Delta P}{2L}(y_1 + y_2) = \frac{\Delta P}{L} d \tag{7.81}$$

Therefore, the shear stress in region 2 is given by

$$\begin{aligned} \tau_2(y) &= -\frac{\Delta P}{L} y + \frac{\Delta P}{2L} d \\ &= \frac{\Delta P}{2L}(d - 2y) \end{aligned} \tag{7.82}$$

The plug thickness can be found by substituting the constant C_5 in the first boundary condition (Eq. 7.79)

$$-\frac{\Delta P}{L} y_1 + \frac{\Delta P}{2L} d = \tau_y \implies \delta = \tau_y \frac{2L}{\Delta P} \tag{7.83}$$

It is convenient to nondimensionalize the plug thickness by the height of the gap

$$\bar{\delta} = \frac{\delta}{d} = \frac{\tau_y 2L}{\Delta P d} \tag{7.84}$$

The value of $\bar{\delta}$ defines the state of flow through the gap.

(1) $\bar{\delta} = 0$: The flow is purely Newtonian.
(2) $\bar{\delta} = 1$: The gap is completely blocked and there is no flow of fluid. Given a specific fluid, the differential pressure below which the flow passage remains blocked can be derived as

$$\Delta P \leq \frac{2\tau_y L}{d} \tag{7.85}$$

Alternatively, to sustain a specified pressure differential without allowing any flow, a fluid can be chosen with a yield stress such that

$$\tau_y \geq \frac{\Delta P d}{2L} \tag{7.86}$$

To calculate the effective damping coefficient of the activated fluid element, it is necessary to find a mean flow velocity, u_m, by finding the total volumetric flow Q through the passage

$$\begin{aligned} Q &= \int_{y=0}^{d} u(y)\, b\, dy \\ &= 2\, Q_1 + Q_2 \end{aligned} \tag{7.87}$$

where Q_1 and Q_2 is the volumetric flow through region 1 and region 2, respectively, given by (from Eqs. 7.66 and 7.75)

$$Q_1 = b\int_0^{y_1} \frac{\Delta P}{2\mu L}\left(2yy_1 - y^2\right) dy = \frac{\Delta P b}{24\mu L}(d-\delta)^3 \tag{7.88}$$

$$Q_2 = b\int_{y_1}^{y_2} u_p\, dy = \frac{\Delta P b}{8\mu L}(d-\delta)^2\delta \tag{7.89}$$

Note that $Q_3 = Q_1$. The total volumetric flow is given by

$$\begin{aligned} Q &= u_m b\, d \\ &= \frac{\Delta P b}{12\mu L}(d-\delta)^3 + \frac{\Delta P b}{8\mu L}(d-\delta)^2\delta \\ &= \frac{\Delta P b}{12\mu L}(d-\delta)^2\left(d+\frac{\delta}{2}\right) \\ &= \frac{\Delta P b\, d^3}{12\mu L}(1-\bar{\delta})^2\left(1+\frac{\bar{\delta}}{2}\right) \end{aligned} \tag{7.90}$$

From this equation, the mean velocity can be extracted as

$$u_m = \frac{\Delta P d^2}{12\mu L}(1-\bar{\delta})^2\left(1+\frac{\bar{\delta}}{2}\right) \tag{7.91}$$

The damping coefficient c_{eq}^a in the active state is given by

$$c_{eq}^a = \frac{F_a}{u_m} \tag{7.92}$$

where F_a is the force required to move the piston when the fluid is activated, given by

$$F_a = \Delta P\, bd \tag{7.93}$$

From these equations, the active damping coefficient is

$$c_{eq}^a = \frac{12\mu L b d}{d^2(1-\bar{\delta})^2(1+\bar{\delta}/2)} = \frac{c_{eq}^o}{(1-\bar{\delta})^2(1+\bar{\delta}/2)} \tag{7.94}$$

The ratio of the damping coefficient in the active state to the damping coefficient in the inactive state, as a function of different plug thicknesses, is shown in Fig. 7.28. It can be seen that this ratio increases steeply as the plug thickness increases. For a plug thickness of around 0.6, the damping coefficient increases by an order of magnitude from the inactive to the active state.

The ratio of the damping coefficients in the active and inactive states can also be expressed in terms of the Bingham number. The Bingham number is defined as

$$\begin{aligned} \mathrm{Bi} &= \frac{\tau_y d}{\mu u_m} \\ &= \frac{\tau_y d}{\mu}\frac{12\mu L}{\Delta P d^2(1-\bar{\delta})^2(1+\bar{\delta}/2)} \end{aligned} \tag{7.95}$$

From the definition of plug thickness (Eq. 7.84)

$$\mathrm{Bi} = \frac{6\bar{\delta}}{(1-\bar{\delta})^2(1+\bar{\delta}/2)} \tag{7.96}$$

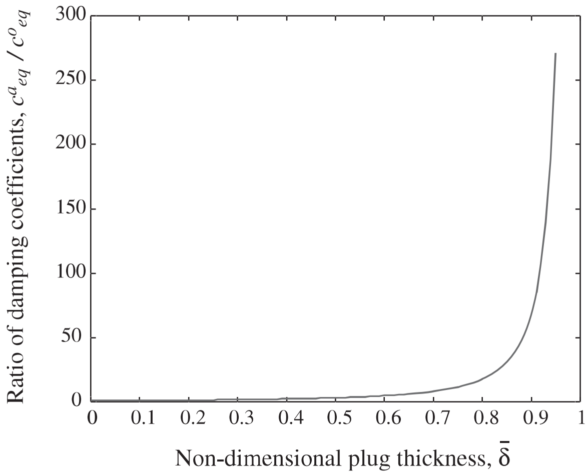


Figure 7.28. Variation of the ratio of damping coefficients with plug thickness.

which yields the ratio of damping coefficients as

$$\frac{c^a_{eq}}{c^o_{eq}} = \frac{\mathrm{Bi}}{6\bar{\delta}} \tag{7.97}$$

It is interesting to note that using the Bingham plastic model, it is possible to obtain a value of $\bar{\delta} = 1$, meaning fully blocked flow. This would yield a damping coefficient of infinity, which is not realistic. Using the biviscous fluid model would alleviate this problem because of the finite pre-yield viscosity.

Piston Area and Flow Passage Area

Often, for flow mode dampers, the cross-sectional area of the piston head (A_p) may not be the same as the cross-sectional area of the flow passage (A_d). An example of such a case is a bypass damper (shown in Fig. 7.29). In this case, the damping coefficient calculated from the force and velocity in the flow passage is different from the damping coefficient with respect to the force and velocity of the piston. The volume of fluid displaced by the piston head is given by

$$Q_p = A_p u_p \tag{7.98}$$

where u_p is the velocity of the piston head. The effective damping coefficient of the bypass damper, c_{eq}, is defined with respect to the piston velocity and the force on the piston, F_p

$$F_p = c_{eq} u_p \tag{7.99}$$

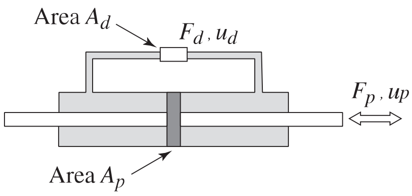


Figure 7.29. Equivalent damping coefficient of a bypass damper.