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Nonholonomic Kinematics and Dynamics of the Sphericle

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Abstract

In this paper we consider a complete dynamic model for the "Sphericle", a spherical vehicle that has been designed and realized in our laboratory. The sphericle is able to roll on the floor of the laboratory and reach arbitrary positions and orientations, through the use of only two motors placed within the rolling sphere. In this paper, we report on the derivation of the kinematic model of the Sphericle, which incorporates two types of nonholonomic constraints, and its dynamic model.

1 Introduction

In recent years, the study of systems with nonholonomic constraints has attracted a lot of attention for several reasons (see e.g. [1]). Such constraints arise naturally in many mechanical devices: typical cases are car-like vehicles [2, 3], underwater vehicles, underactuated satellites, or dexterous robotic hands [4, 5, 6, 7, 8, 9, 10, 11, 12]. Sometimes nonholonomic constraints are introduced on purpose to obtain a better behaviour of the system. An interesting aspect of such systems is due to the fact that they need a smaller number of actuators than the number of independent configurations at equilibrium; this imply a lower complexity (and cost) of the mechanical system. On the other hand, nonholonomic systems, introduce many difficulties in the analysis and control of the system. The dynamic modeling is more complicated than for unconstrained or holonomically constrained systems. Control design, on the other hand, cannot afford the powerful results of linear systems theory (the linear approximation of nonholonomic system causes the loss of controllability), and it is well known by now that nonholonomic systems can not be stabilized via continuous feedback control laws.

In this paper we describe an experimental apparatus developed in our laboratory for research and advanced teaching purposes, previously introduced in [13]. The kinematics of the vehicle are nonholonomic, and result from the combination of the kinematics of two classical nonholonomic systems, namely, a unicycle and a plate-ball system. The "sphericle" is a ball that rolls freely on the floor, and can reach any arbitrary position therein and orientation (as shown in [13]). To make the ball move, a mobile mass is placed

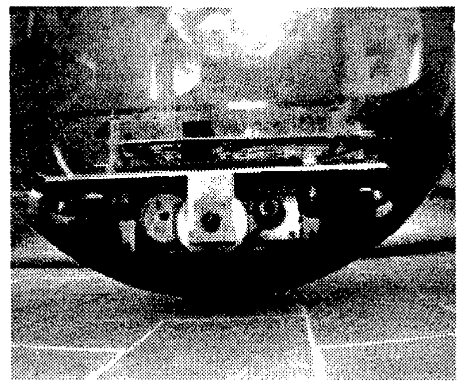

Figure 1: A transparent version of the Sphericle shows the inner moving mass with wheels and suspensions.

within the cavity of the ball. To implement motion, we built a mechanism with an inner mass moving by means of two weels (differential drive), which roll on the internal surface of the sphere. The inner vehicle is inserted through an opening in the ball, which is sealed afterwards.

In section 2, the kinematics of the Sphericle are derived using a different approach than was used in [13]. A dynamic model of the vehicle is then considered, which uses Euler–Lagrange equations in quasi–coordinates (section 3).

2 Kinematic Modeling

The kinematic model of the Sphericle (see fig.1) will be derived in the assumption that the sphere rolls without slipping on the floor. Moreover, rotations of the sphere around the vertical axis are not allowed. The ball thickness is neglected, and the projection of the center of the inner vehicle is always considered to coincide with the contact point between the sphere and the floor.

To describe the sphere and inner mass position, choose the coordinates x and y of the contact point between the sphere and the floor, in an orthogonal

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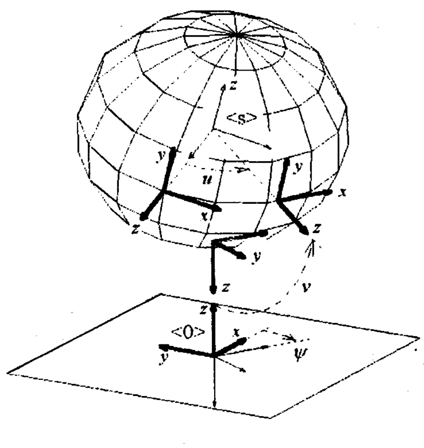

Figure 2: Lagrangian coordinates for the sphere: x, y, u, v, ψ.

reference frame fixed to the floor; if θ denotes the inner mass orientation, we get $\dot{x} = \cos\theta u_1$, $\dot{y} = \sin\theta u_1$, $\dot{\theta} = u_2$, i.e., the classical unicycle kinematics equations, where u_1 is the forward velocity of the inner mass, i.e. the average of the linear velocity of the wheels, and $\mathbf{u}_2$ is the angular velocity of the inner mass, i.e. the difference of the linear velocity of the wheels.

Three variables describing the sphere orientation are the azimuth u and elevation v of the contact point between the sphere and the floor in a spherical coordinate reference frame fixed to the ball, and the holonomy angle ψ between the x–axes of the floor and ball Gauss frames at the contact point (see fig.2), so that the rotation matrix R_s between the sphere frame and the inertial frame (whose columns are the sphere frame axes projected on the inertial frame) is

$$R_s(u, v, \psi) = R_x(\pi)\ R_z(\psi)\ R_x(v)\ R_y(-u) \qquad (1)$$

where R_x, R_y and R_z are the elementary rotations (the constant matrix $R_x(\pi)$ is due to the fact that the Gauss frames have opposite z–axes).

The Jacobian matrix that relates the Lagrangian velocities $\dot{u}$, $\dot{v}$ and $\dot{\psi}$ with the sphere angular velocity ω_s, i.e.

$$\omega_s = J_s \frac{d}{dt}[u, v, \psi]^T \qquad (2)$$

is

$$J_s(u, v, \psi) = [-R_s j \mid R_x(\pi)R_z(\psi)R_x(v)i \mid R_x(\pi)R_z(\psi)k] = \begin{bmatrix} \cos v \sin\psi & \cos\psi & 0 \\ \cos v \cos\psi & -\sin\psi & 0 \\ \sin v & 0 & -1 \end{bmatrix}$$

where $i = [1,0,0]^T$, $j = [0,1,0]^T$ and $k = [0,0,1]^T$. The description 1 is not globally valid, as $v = \pm\pi/2$ is singularity point for the chosen coordinates (in fact $det(J_s) = \cos v$). A suitable change of coordinates should be applied when in a neighborhood of singularities.

The system variables are

$$q = [x,\ y,\ u,\ v,\ \psi,\ \theta]^T.$$

We define $v_{Px} := \dot{x} - \rho\omega_{sy}$ and $v_{Py} := \dot{y} + \rho\omega_{sx}$ the velocities of the point of the sphere in contact with the plane. Since the nonholonomic constraints for the system are

$$\begin{cases} v_{Px} = 0 \\ v_{Py} = 0 \\ \omega_{sz} = 0 \\ \frac{\dot{y}}{\dot{x}} = \frac{\sin\theta}{\cos\theta} \end{cases}$$

(respectively the sphere rolls without slip, the sphere don't spin and the direction is constrained by the inner mass wheels), the following equation describes the sphericle kinematics

$$\dot{q} = g_1(q)u_1 + g_2(q)u_2, \qquad (3)$$

where

$$g_1(q) = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{\cos(\theta+\psi)}{\rho\cos v} \\ -\frac{\sin(\theta+\psi)}{\rho} \\ \frac{\tan v \cos(\theta+\psi)}{\rho} \\ 0 \end{bmatrix}, \quad g_2(q) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The system has two inputs (u_1 and u_2, as already defined) and six states.

3 Dynamic Modeling

In this section the dynamic equations for the Sphericle are derived applying the Lagrangian approach in quasi coordinates. Recall that the Lagrange equation in nonholonomically constrained, generalized coordinates is written as

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} + A^T\lambda = Fu \qquad (4)$$

where q is a set of independent variable that describes the system configuration, the Lagrangian $L(q, \dot{q})$ is the difference of the kinetic energy and potential energy i.e. $L(q, \dot{q}) = T(q, \dot{q}) - V(q)$, $F(q)$ is a $n \times m$ full rank matrix, the vector of the generalized external forces u is the control, and A is an $k \times n$ matrix that represents the k nonholonomic constraints in the so called Pfaffian form (i.e. the velocities appear linearly in the matrix equations):

$$A(q)\dot{q} = 0. \qquad (5)$$

The resulting equations can be written in matrix form

$$M(q)\ddot{q} + N(q,\dot{q}) + A^T(q)\lambda = F(q)u \qquad (6)$$
$$A(q)\dot{q} = 0 \qquad (7)$$

where $M(q)$ is the $n \times n$ symmetric inertia matrix, and the term $A^T(q)\lambda$ represents the generalized constraint forces. If we consider the $n \times (n-k)$ full rank matrix $S(q)$ such that $A(q)S(q) = 0$, we can eliminate the unknown Lagrange multipliers λ pre-multiplying the eq. (6) by $S^T(q)$, and then we can eliminate the equation (7) using a set of $n-k$ independent velocities ν, such that

$$\dot{q} = S(q)\nu. \qquad (8)$$

Finally we obtain a system in the form

$$\bar{M}(q)\dot{\nu} + \bar{N}(q,\nu) = \bar{F}(q)u \qquad (9)$$

where $\bar{M}(q)$ is a $(n-k) \times (n-k)$ symmetric matrix, $\bar{N}(q,\nu)$ and $\bar{F}(q)$ are respectively a $(n-k)$ vector and a $(n-k) \times m$ matrix.

The problem with applying such classical derivation to the sphericle dynamics is that the expression of kinetic and potential energy in terms of generalized coordinates for the system is quite cumbersome. In systems with nonholonomic constraints, in order to simplify the energetic description and to obtain directly the equation in the form (9), it is expedient to define the kinetic energy by means of a set of nonintegrable velocities instead of the time derivatives of the Lagrangian variables $\dot{q}$. We can thus include the constraints in the Lagrange equation (4) before the calculus of derivatives. In fact, if we choose the set $\{\nu, \bar{\nu}\}$, with ν defined in (8) and $\bar{\nu}$ such that the nonholonomic constraints become simply $\bar{\nu} = 0$, using (5), we obtain

$$\begin{bmatrix} \nu \\ \bar{\nu} \end{bmatrix} = \begin{bmatrix} S^+(q) \\ A(q) \end{bmatrix} \dot{q}.$$

The Lagrange equation in quasi–coordinates evaluates to

$$\left| \frac{d}{dt}\frac{\partial L}{\partial \nu} - S^T \frac{\partial L}{\partial q} + S^T \Gamma S \nu \right|_{\bar{\nu}=0} = \bar{F}u \qquad (10)$$

where $\Gamma(q,\nu,\bar{\nu})$ is the skew symmetric matrix defined by

$$\begin{aligned} \Gamma_{i,j} &= \frac{\partial T}{\partial \nu}\left(\frac{\partial S^+_{col\ i}}{\partial q_j} - \frac{\partial S^+_{col\ j}}{\partial q_i}\right) \\ &+ \frac{\partial T}{\partial \bar{\nu}}\left(\frac{\partial A_{col\ i}}{\partial q_j} - \frac{\partial A_{col\ j}}{\partial q_i}\right) \end{aligned} \qquad (11)$$

Notice that, wherever the derivatives with respect to the constrained velocities, namely $\frac{\partial T}{\partial \bar{\nu}}$, need not be computed, we can use a simplified form for the kinetic energy $\bar{T}(q,\nu) := T(q,\nu,0)$ (i.e. we include the nonholonomic constraints).

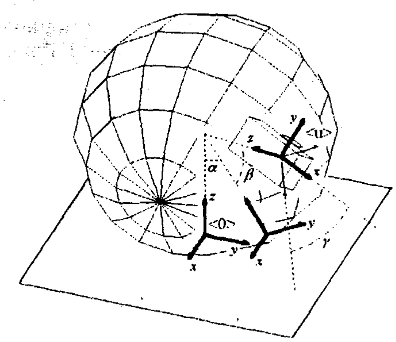

Figure 3: Lagrangian coordinates for the inner mass: α, β, and γ.

In order to compute equation (10), first define the configuration variables q to include angles α, β, and γ describing respectively the roll, pitch and yaw angles of the inner mass with respect to a frame fixed to the floor (see fig.3), so that the rotation matrix R_u between the inner mass frame and the inertial frame (the R_u columns are the inner mass frame axes projected on the inertial frame) is given by

$$R_u(\alpha,\beta,\gamma) := R_x(\alpha)\ R_y(\beta)\ R_z(\gamma) =$$

$$= \begin{bmatrix} c_\beta c_\gamma & -c_\beta s_\gamma & s_\beta \\ s_\alpha s_\beta c_\gamma + c_\alpha s_\gamma & -s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma & -s_\alpha c_\beta \\ -c_\alpha s_\beta c_\gamma + s_\alpha s_\gamma & c_\alpha s_\beta s_\gamma + s_\alpha c_\gamma & c_\alpha c_\beta \end{bmatrix}$$

where $s_\alpha = sin(\alpha)$, $c_\alpha = cos(\alpha)$ and so on. The Jacobian matrix that relates the Lagrangian variables with the inner mass angular velocities ω_u, i.e.

$$\omega_u = J_u \frac{d}{dt}[\alpha,\beta,\gamma]^T \qquad (12)$$

is

$$J_u(\alpha,\beta,\gamma) = \begin{bmatrix} 1 & 0 & sin\beta \\ 0 & \cos\alpha & -\sin\alpha\cos\beta \\ 0 & \sin\alpha & \cos\alpha\cos\beta \end{bmatrix}$$

The other Lagrangian variables remains the same of the quasi-static model, so the configuration variables are described by the 8–dimensional vector

$$q = [x\ \ y\ \ u\ \ v\ \ \psi\ \ \alpha\ \ \beta\ \ \gamma]^T.$$

The singularity points for the chosen coordinates are $v = \pm\pi/2$ and $\beta = \pm\pi/2$. The system has 4 degrees of freedom: the angular velocity of the sphere with respect to the inertial frame ω_{sx} and ω_{sy} (the rotation around the vertical axis ω_{sz} is not allowed), and the angular velocity of the inner mass with respect to the sphere, projected on the frame fixed to the inner mass: ω_{uy} and ω_{uz} ($\omega_{ux} = 0$ because of the nonholonomic constraint due to the wheels). So the 4–dimensional independent velocity vector is

$$\nu := [\omega_{sx} \;\; \omega_{sy} \;\; \omega_{uy} \;\; \omega_{uz}]^T \tag{13}$$

while the constrained velocity could be chosen as

$$\tilde{\nu} := [v_{Px} \;\; v_{Py} \;\; \omega_{sz} \;\; \omega_{ux}]^T = 0 \tag{14}$$

where, as already defined, $v_{Px} = \dot{x} - \rho\omega_{sy}$ and $v_{Py} = \dot{y} + \rho\omega_{sx}$ are the velocity of the point of the sphere in contact with the plane. Notice that each item of $\tilde{\nu}$ describes a nonholonomic constraint; therefore, by definition, $\tilde{\nu}$ is not integrable. The 4×8 constraints matrix, defined in (5) is

$$A(q) = \begin{bmatrix} I_{2\times 2} & \rho \begin{bmatrix} -j^T \\ i^T \end{bmatrix} J_s & 0_{2\times 3} \\ 0_{1\times 2} & -i^T R_u^T J_s & i^T R_u^T J_u \\ 0_{1\times 2} & k^T J_s & 0_{1\times 3} \end{bmatrix}$$

while, using definition 13, we obtain the 8×4 matrix (defined in (8)) that relates the independent velocity with the (non-independent) Lagrangian variables derivatives, i.e. $\dot{q} = S(q)\nu$:

$$S(q) := \begin{bmatrix} \rho \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} & 0_{2\times 2} \\ J_s^{-1} [\; i \;\; j \;] & 0_{3\times 2} \\ J_u^{-1} [\; i \;\; j \;] & J_u^{-1} R_u [\; j \;\; k \;] \end{bmatrix}$$

Obviously it holds $A(q)S(q) = 0$. The potential energy of the sphere is clearly constant, so $V(q)$ depends only on the inner mass position:

$$V(q) = -M_u g(k \cdot r_u) = -M_u g \rho \cos q_6 \cos q_7$$

where $r_u = r_s - \rho R_u k$ is the inner mass position, $r_s = [x, y, \rho]^T$ is the sphere position, M_u is the inner mass mass, g is the gravitational acceleration. The kinetic energy is

$$\begin{aligned} T(q, \dot{q}) &= \frac{M_s}{2} \left\| \frac{dr_s}{dt} \right\|^2 + \frac{1}{2}\omega_s^T I_s \omega_s \\ &+ \frac{M_u}{2} \left\| \frac{dr_u}{dt} \right\|^2 + \frac{1}{2}\omega_u^T I_u \omega_u \end{aligned} \tag{15}$$

where ω_s is the sphere angular velocity, and ω_u can be written using eq. (12). M_s is the sphere mass, I_s and I_u are respectively the inertia matrix of the sphere and the inner mass; with no loss of generality we suppose I_u is a diagonal matrix, i.e. $I_u = diag(I_{ux}, I_{uy}, I_{uz})$. Because of the sphere symmetry, I_s is proportional to the identity matrix, so in the further we refer I_s as a scalar; notice that for a small tickness of the ball surface it results $I_s \simeq \frac{2}{3} M_s \rho^2$.

A simple choice for the pseudo–inverse $S^+(q)$, such that $\nu = S^+(q)\dot{q}$ is

$$S^+(q) = \begin{bmatrix} 0_{2\times 2} & \begin{bmatrix} i^T \\ j^T \end{bmatrix} J_s & 0_{2\times 3} \\ 0_2 & -\begin{bmatrix} j^T \\ k^T \end{bmatrix} R_u^T J_s & \begin{bmatrix} j^T \\ k^T \end{bmatrix} R_u^T J_u \end{bmatrix}$$

The system input in eq. (10) is represented by $\bar{F}u$; as only the inner mass is actuated, using definition 13, we choose

$$\bar{F} := \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \rho & 0 \\ 0 & 1 \end{bmatrix}, \quad u := \begin{bmatrix} u_f \\ u_\tau \end{bmatrix}$$

where u_f is the forward force applied by the inner mass wheels (i.e. the average of the torque of the wheels divided by the wheel radius) and u_τ is the rotational torque applied by the actuators (proportional to the difference of the wheels torque).

3.1 Analytic Results

Summarizing, the dynamics of the sphericle can be written in the form

$$\bar{M}(q)\dot{\nu} + \bar{n}'(q,\nu) + \bar{n}''(q) = \bar{F}u$$

where

$$M(q) = \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} & I_{uz}s_7 \\ M_{2,1} & M_{2,2} & M_{2,3} & -I_{uz}s_6c_7 \\ M_{3,1} & M_{3,2} & M_u\rho^2 & 0 \\ I_{uz}s_7 & -I_{uz}s_6c_7 & 0 & I_{uz} \end{bmatrix}$$

$$\begin{aligned} M_{1,1} &= M_s\rho^2 + I_{sx} + M_u\rho^2(1 - 2c_6c_7 + c_7^2) + I_{uz}s_7^2 \\ M_{1,2} &= M_{2,1} = (M_u\rho^2 - I_{uz})s_7s_6c_7 \\ M_{1,3} &= M_{3,1} = M_u\rho^2(c_6s_8 - s_8c_7 + s_7c_8s_6) \\ M_{2,2} &= M_s\rho^2 + I_{sx} + M_u\rho^2(2 - s_6^2c_7^2 - 2c_6c_7) + I_{uz}s_6^2c_7^2 \\ M_{2,3} &= M_{3,2} = M_u\rho^2(-c_7c_8 + c_6c_8 - s_8s_6s_7) \end{aligned}$$

The vector $\bar{n}'(q,\nu) = [\bar{n}'_1,\; \bar{n}'_2,\; \bar{n}'_3,\; \bar{n}'_4]^T$ is defined by

$$\bar{n}'_i(q,\nu) := \nu^T N_i(q)\nu$$

where

$$\begin{aligned} N_{i(j,k)} &= 0 \quad for \;\; j >= k \\ N_{1(1,2)} &= M_u\rho^2 s_7 - (M_u\rho^2 - I_{vz})s_7c_6c_7 \\ N_{1(1,3)} &= 2M_u\rho^2(c_6s_7c_8 - s_6s_8) - 2(M_u\rho^2 - I_{uz})c_7c_8s_7 \\ N_{1(2,3)} &= 2(M_u\rho^2 - I_{uz})s_6c_7^2c_8 + I_{uz}(s_8c_6s_7 + c_8s_6) \\ N_{1(2,4)} &= -N_{2(1,4)} = I_{uz}c_6c_7 \\ N_{1(3,4)} &= M_u\rho^2(c_6c_8 - s_8s_6s_7) - (M_u\rho^2 - I_{uz})c_7c_8 \\ N_{2(1,2)} &= M_u\rho^2 s_6c_7 - (M_u\rho^2 - I_{uz})s_6c_6c_7^2 \\ N_{2(1,3)} &= -M_u\rho^2(c_6s_7s_8 + s_6c_8)+ \\ &\quad +(M_u\rho^2 - I_{uz})(2s_6c_7^2c_8 - c_6s_7s_8 - s_6c_8) \\ N_{2(2,3)} &= 2M_u\rho^2(-s_6s_8 + c_6s_7c_8)+ \\ &\quad +2(M_u\rho^2 - I_{uz})(c_7s_8c_6s_6 - c_6^2c_7s_7c_8 + c_7s_7c_8) \\ N_{2(3,4)} &= M_u\rho^2 s_8c_7 - (M_u\rho^2 - I_{uz})(c_6s_8 + s_7c_8s_6) \\ N_{3(1,2)} &= (M_v\rho^2 - I_{uz})(s_6c_8 + c_6s_7s_8 - 2s_6c_8c_7^2) \\ N_{3(1,4)} &= -N_{4(1,3)} = -I_{uz}c_8c_7 \\ N_{3(2,4)} &= -N_{4(2,3)} = -I_{uz}(s_7c_8s_6 + c_6s_8) \\ N_{1(1,4)} &= N_{2(2,4)} = N_{3(1,3)} = N_{3(2,3)} = N_{3(3,4)} = \\ &= N_{4(1,2)} = N_{4(1,4)} = N_{4(2,4)} = N_{4(3,4)} = 0 \end{aligned}$$

The the gravitational force vector $\bar{n}''(q)$ is given by

$$\bar{n}''(q) = g\rho M_u \begin{bmatrix} s_6 c_7 \\ s_7 \\ c_6 s_7 c_8 - s_6 s_8 \\ 0 \end{bmatrix}$$

3.2 Linearized Longitudinal Dynamics

In this section, we focus on the the longitudinal dynamics of the sphericle, corresponding to rolling in the forward direction of the inner mass, without steering, and neglecting the (nonoholonomic) coupling between lateral oscillations and longitudinal motion. To obtain such approximate decoupling between longitudinal and lateral dynamics, it is expedient to consider a linearized model of the sphericle. Although such linearization will undoubtedly destroy controllability, it provides a good approximated decoupled model for the longitudinal dynamics (which are holonomic). The linearized equations of the sphericle system are

$$\dot{x} = Ax + Bu \qquad (16)$$

Defining

$$x := \begin{bmatrix} q \\ \nu \end{bmatrix} \qquad (17)$$

($dim(x) = 12$) and if choosing as equilibrium point the state $x = 0$, we get

$$A = \left[\begin{array}{ccc|cccc} & 0_{8\times 8} & & \begin{matrix} 0 & \rho & 0 & 0 \\ -\rho & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \\ \hline 0_{1\times 5} & \begin{matrix} -H & 0 & 0 \\ 0 & -H & 0 \\ 0 & \frac{-g}{\rho} & 0 \\ 0 & 0 & 0 \end{matrix} & & 0_{4\times 4} \end{array}\right]$$

were $H := \frac{M_u \rho g}{I_s + M_s \rho^2}$, and

$$B = \begin{bmatrix} 0_{10\times 2} & \\ -\frac{1}{M_u \rho} & 0 \\ 0 & \frac{1}{I_{uz}} \end{bmatrix};$$

it results

$$\mathsf{rank}\,([B|AB|\dots|A^{11}B]) = 6.$$

As the system has an intrinsic tendency to oscillate, in order to ensure the validity of the quasi-static kinematic model used for steering (see sect. 2), a feedback control law that reduces any undesired oscillations is needed. Although to obtain the controller in general is an open problem, we consider here a 4–dimensional reduced model which is obtained by projecting the states on the plane perpendicular to the axle of the wheels (see fig.4). This results in an independent controllable subsystem of the linearized model. Naturally, feedback laws designed for such subsystem will not control rolling oscillations. However, maneuvers such as accelerating in the longitudinal direction, and slow steering, typically will generate small such oscillations, which will in turn affect the longitudinal dynamics (through nonholonomic coupling) negligibly.

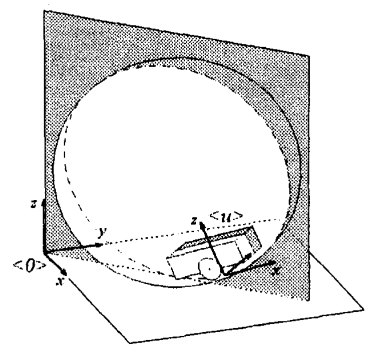

Figure 4: Projection of the states on a vertical plane containing the inner mass direction vector.

Let us consider the two dimensional reduced model of longitudinal dynamics in fig.5. In this case, it is convenient to change the definition of the state describing the inner mass position: while in the three dimensional model the inner mass position is projected on the inertial frame, here we use the inner mass position with respect to the sphere position (see α in fig.5). Defining

$$\xi := [\theta,\ \alpha,\ \dot{\theta},\ \dot{\alpha}]^T,$$

and using definition (17), we obtain $\{\xi_1, \xi_2, \xi_3, \xi_4\} := \{x_3, x_3 - x_7, x_{10}, -x_{12}\}$.

With respect to the model described above, we introduce few modifications for the particular implementation of the sphericle. In particular, because the inner moving mass is actuated in our prototype by stepper motors, instead of considering torques, we will take as inputs the derivatives of the pulse rates commanded at the motors (i.e., up to neglecting quantization errors, angular accelerations of the wheels). Also, we consider explicitly the distance h along the radial direction of the center of mass of the inner vehicle from the sphere surface, and include a frictional dissipation torque $-\mu_\theta \dot{\theta}$. The corresponding linearized model of the longitudinal dynamics is

$$\dot{\xi} = A\xi + Bu \qquad (18)$$

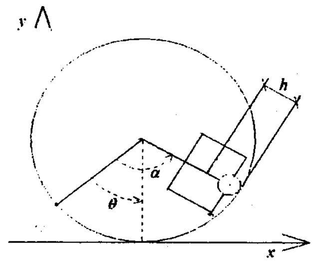

Figure 5: The two dimensional model. Notice that $x_{sphere} = \rho\theta$.

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{M_u(\rho-h)}{H}g & \frac{M_u(\rho-h)}{H}g & \frac{\mu_\theta}{H} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{-M_u h(\rho-h)+I_{uy}}{H} \\ 1 \end{bmatrix}$$

with $H := M_s\rho^2 + M_u h^2 + I_s + I_{uy}$. The subsystem (18) is completely controllable. Measurements available for this system are $\dot{\alpha}$, α (by odometry) and the pitch angle $\alpha - \theta$ using an inclinometer. With these outputs, system (18) is also observable.

4 Conclusion

We have obtained a complete dynamic model for the Sphericle, an interesting nonholonomic system for locomotion. Dynamics have been computed in closed form through use of the Euler–Lagrange equations in quasi-coordinates, a method which fits well nonholonomic dynamics. A simplified linear model of the longitudinal dynamics of the vehicle have been reported. These models are fundamental tools for implementing control laws for the vehicle. At the moment of writing, no general control law for the sphericle is available, while results on the stabilization of the longitudinal dynamics were obtained by the authors that applied satisfactorily to the experimental device, but are not reported here because of space limitations.

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